Optimization approaches to multiplicative tariff of rates estimation in non-life insurance

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Four methodologies

The contribution combines four methodologies:

- **Data-mining** – data preparation.
- **Mathematical statistics** – random distribution estimation using generalized linear models.
- **Insurance mathematics** – pricing of non-life insurance contracts.
- **Operations research** – (stochastic) optimization approach to tariff of rates estimation based on the previous methodologies.

Practical experiences

- More than 3 years at Actuarial Department, Head Office of Vienna Insurance Group Czech Republic.
- VIG CR – the largest group on the market: 2 universal insurance companies (Kooperativa pojišťovna, Česká podnikatelská pojišťovna) and 1 life-oriented (Česká spořitelna).
- Kooperativa & ČPP MTPL: 2.5 mil. cars from 7 mil.
- Kooperativa & ČPP: common back-office (data-warehouse, data-mining).
- Kooperativa & ČPP: completely different portfolios and strategies, e.g. flat MTPL rates vs. strict segmentation.

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Tariff classes/segmentation criteria

Tariff of rates based on $S+1$ **segmentation criteria**:

- $i_0 \in \mathcal{I}_0$, e.g. tariff classes $\mathcal{I}_0 = \{$engine up to 1000, up to 1350, up to 1850, up to 2500, over 2500 ccm$\}$,
- $i_1 \in \mathcal{I}_1, \dots, i_S \in \mathcal{I}_S$, e.g. age $\mathcal{I}_1 = \{$18–30, 31–65, 66 and more years$\}$

We denote $I = (i_0, i_1, \dots, i_S)$, $I \in \mathcal{I}$ a **tariff class**, where $\mathcal{I} = \mathcal{I}_0 \otimes \mathcal{I}_1 \otimes \cdots \otimes \mathcal{I}_S$ denotes all combinations of criteria values. Let W_I be the number of contracts in I.

Compound distribution of aggregated losses

Aggregated losses over one year for risk cell I

$$L_I^T = \sum_{w=1}^{W_I} L_{I,w}, \;\; L_{I,w} = \sum_{n=1}^{N_{I,w}} X_{I,n,w},$$

where all r.v. are assumed to be independent (N_I, X_I denote independent copies)

- $N_{I,w}$ is the random **number of claims** for a contract during one year with the same distribution for all w
- $X_{I,n,w}$ is the random **claims severity** with the same distribution for all n and w

Well-known formulas for the mean and the variance:

$$\begin{aligned} \mu_I^T &= \mathbb{E}[L_I^T] = W_I \mu_I = W_I \mathbb{E}[N_I]\mathbb{E}[X_I], \\ (\sigma_I^T)^2 &= var(L_I^T) = W_I \sigma_I^2 = W_I(\mathbb{E}[N_I] var(X_I) + (\mathbb{E}[X_I])^2 var(N_I)). \end{aligned}$$

Multiplicative tariff of rates

We assume that the risk (office) premium is composed in a multiplicative way from

- **basic premium levels** Pr_{i_0} and
- nonnegative **surcharge coefficients** $e_{i_1}, \dots, e_{i_S}$,

i.e. we obtain the decomposition

$$Pr_I = Pr_{i_0} \cdot (1 + e_{i_1}) \cdot \dots \cdot (1 + e_{i_S}).$$

We denote the **total premium** $TP_I = W_I Pr_I$ for the risk cell I.

Example: engine between 1001 and 1350 ccm, age 18–30, region over 500 000:

$$130 \cdot (1 + 0.5) \cdot (1 + 0.4)$$

Prescribed loss ratio – random constraints

Our goal is to find optimal basic premium levels and surcharge coefficients with respect to a **prescribed loss ratio** $\hat{LR}$, i.e. to fulfill the **random constraints**

$$\frac{L_I^T}{TP_I} \leq \hat{LR} \text{ for all } I \in \mathcal{I}, \tag{1}$$

and/or the **random constraint**

$$\frac{\sum_{I \in \mathcal{I}} L_I^T}{\sum_{I \in \mathcal{I}} TP_I} \leq \hat{LR}. \tag{2}$$

The prescribed loss ratio $\hat{LR}$ is usually based on a management decision. If $\hat{LR} = 1$, we obtain the netto-premium. It is possible to prescribe a different loss ratio for each tariff cell.

Sources of risk

Two sources of risk:

1. **Expectation risk**: different **expected losses** for tariff cells.
2. **Distributional risk**: different **shape of the probability distribution** of losses, e.g. standard deviation.

Prescribed loss ratio – expected value constraints

Usually, the **expected value** of the loss ratio is bounded

$$\frac{\mathbb{E}[L_l^T]}{TP_l} = \frac{\mathbb{E}[L_l]}{Pr_l} \leq \hat{LR} \text{ for all } l \in \mathcal{I}. \tag{3}$$

The distributional risk is not taken into account.

Prescribed loss ratio – chance constraints

A natural requirement: the inequalities are fulfilled with a **prescribed probability** leading to individual chance (probabilistic) constraints

$$P\left(\frac{L_l^T}{TP_l} \leq \hat{LR}\right) \geq 1 - \varepsilon, \text{ for all } l \in \mathcal{I}, \tag{4}$$

where $\varepsilon \in (0, 1)$, usually $\varepsilon \in \{0.1, 0.05, 0.01\}$, or a constraint for the whole line of business:

$$P\left(\frac{\sum_{l\in\mathcal{I}} L_l^T}{\sum_{l\in\mathcal{I}} TP_l} \leq \hat{LR}\right) \geq 1 - \varepsilon.$$

Distributional risk allocation to tariff cells will be discussed later.

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Generalized linear models

A standard approach based on GLM with the **logarithmic link function** $g(\mu) = \ln \mu$ without the intercept:

- **Poisson (overdispersed) or Negative-binomial regression** – the expected number of claims:

$$\mathbb{E}[N_I] = \exp\{\lambda_{i_0} + \lambda_{i_1} + \cdots + \lambda_{i_S}\},$$

- **Gamma or Inverse Gaussian regression** – the expected claim severity:

$$\mathbb{E}[X_I] = \exp\{\gamma_{i_0} + \gamma_{i_1} + \cdots + \gamma_{i_S}\},$$

where λ_i, γ_i are the regression coefficients for each $I = (i_0, i_1, \ldots, i_S)$. For the **expected loss** we obtain

$$\mathbb{E}[L_I] = \exp\{\lambda_{i_0} + \gamma_{i_0} + \lambda_{i_1} + \gamma_{i_1} + \cdots + \lambda_{i_S} + \gamma_{i_S}\}.$$

Generalized linear models

The **basic premium levels** and the **surcharge coefficients** can be estimated as a product of normalized coefficients

$$
\begin{aligned}
Pr_{i_0} &= \frac{\exp\{\lambda_{i_0} + \gamma_{i_0}\}}{\hat{LR}} \cdot \prod_{s=1}^{S} \min_{i \in \mathcal{I}_s} \exp(\lambda_i) \cdot \prod_{s=1}^{S} \min_{i \in \mathcal{I}_s} \exp(\gamma_i), \\
e_{i_s} &= \frac{\exp(\lambda_{i_s})}{\min_{i_s \in I_s} \exp(\lambda_{i_s})} \cdot \frac{\exp(\gamma_{i_s})}{\min_{i_s \in I_s} \exp(\gamma_{i_s})} - 1,
\end{aligned}
$$

Under this choice, the constraints on loss ratios are fulfilled with respect to the expectations.

Generalized linear models

The GLM approach is highly dependent on using GLM with the logarithmic link function. It can be hardly used if other link functions are used, interaction or other regressors than the segmentation criteria are considered.

For the total losses modelling, we can employ generalized linear models with the logarithmic link and a **Tweedie distribution** for $1 < p < 2$, which corresponds to the compound Poisson–gamma distributions.

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Advantages of the optimization approach

- GLM with other than logarithmic **link functions** can be used,
- **business requirements** on surcharge coefficients can be ensured,
- total losses can be **decomposed** and modeled using different models, e.g. for bodily injury and property damage,
- **other modelling techniques** than GLM can be used to estimate the distribution of total losses over one year, e.g. generalized additive models, classification and regression trees,
- not only the expectation of total losses can be taken into account but also the **shape of the distribution**,
- **costs and loadings** (commissions, tax, office expenses, unanticipated losses, cost of reinsurance) can be incorporated when our goal is to optimize the combined ratio instead of the loss ratio, we obtain final office premium as the output,

Total loss – decomposition

We can assume that L_I contains not only losses but also various costs and loadings, thus we can construct the tariff rates with respect to a prescribed combined ratio. For example, the **total loss over one year can be composed** as follows

$$L_I = (1 + vc_I)\left[(1 + inf_s)L_I^s + (1 + inf_l)L_I^l\right] + fc_I,$$

where **small** L_I^s and **large** claims L_I^l are modeled separately, **inflation** of small claims inf_s and large claims inf_l, proportional **costs** vc_I and fixed costs fc_I are incorporated.

We only need estimates of $\mathbb{E}[L_I^T]$ and $var(L_I^T)$ for all I.

Optimization model – expected value approach

The premium is minimized[1] under the conditions on the **prescribed loss ratio** and a **maximal possible surcharge** (r^{max}):

$$\begin{aligned}
\min_{Pr,e} \sum_{I\in\mathcal{I}} & w_I Pr_{i_0}(1+e_{i_1})\cdot\dots\cdot(1+e_{i_S}) \\
\text{s.t. } \hat{LR}\cdot Pr_{i_0}\cdot(1+e_{i_1})\cdot\dots\cdot(1+e_{i_S}) & \geq \mathbb{E}[L_{i_0,i_1,\dots,i_S}], \qquad (5)\\
(1+e_{i_1})\cdot\dots\cdot(1+e_{i_S}) & \leq 1+r^{max}, \\
e_{i_1},\dots,e_{i_S} & \geq 0,\ (i_0,i_1,\dots,i_S)\in\mathcal{I}.
\end{aligned}$$

This problem is **nonlinear nonconvex**, thus very difficult to solve. Other constraints can be included.

[1] A profitability is ensured by the constraints on the loss ratio. The optimization leads to minimal levels and surcharges.

Optimization model – expected value approach

Using the **logarithmic transformation** of the decision variables $u_{i_0} = \ln(Pr_{i_0})$ and $u_{i_s} = \ln(1 + e_{i_s})$ and by setting

$$b_{i_0,i_1,\ldots,i_S} = \ln(\mathbb{E}[L_{i_0,i_1,\ldots,i_S}]/\hat{LR}),$$

the problem can be rewritten as a **nonlinear convex programming problem**, which can be efficiently solved by standard software tools:

$$\begin{aligned} \min_u \sum_{I\in\mathcal{I}} & w_I e^{u_{i_0}+u_{i_1}+\cdots+u_{i_S}} \\ \text{s.t. } u_{i_0} + u_{i_1} + \cdots + u_{i_S} & \geq b_{i_0,i_1,\ldots,i_S}, \\ u_{i_1} + \cdots + u_{i_S} & \leq \ln(1 + r^{max}), \\ u_{i_1}, \ldots, u_{i_S} & \geq 0,\ (i_0, i_1, \ldots, i_S) \in \mathcal{I}. \end{aligned} \tag{6}$$

The problems (5) and (6) are equivalent.

Optimization over a net of coefficients

Let the surcharge coefficients be selected from a **discrete net** and $r_s > 0$ be a **step**, usually 0.1 or 0.05. We set $J_s = \lfloor r^{max}/r_s \rfloor$ and

$$u_{i_s} = \sum_{j=0}^{J_s} y_{i_s,j} \ln(1 + j \cdot r_s),$$

together with the conditions $\sum_{j=0}^{J_s} y_{i_s,j} = 1,\ y_{i_s,j} \in \{0, 1\}$.

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Stochastic programming – random right-hand side

The goal is to minimize $f : \mathbb{R}^n \to \mathbb{R}$ under the conditions

$$g_j(x) \geq \xi_j,\ j = 1, \dots, m,$$

where $g_j : \mathbb{R}^n \to \mathbb{R}$ and ξ_j are real random variables. **Chance** (probabilistic, VaR) **constraints**

$$P\left(g_j(x) \geq \xi_j\right) \geq 1 - \varepsilon,\ j = 1, \dots, m,$$

can be reformulated using the quantile function leading to

$$g_j(x) \geq F_{\xi_j}^{-1}(1 - \varepsilon),\ j = 1, \dots, m.$$

A few comments to chance constrained problems

- Chance constraints are **nonconvex** in general.
- It can be even **difficult to verify feasibility** of a points.
- **Solution approaches**:
  - Discrete distribution and mixed-integer programming
  - Sample approximation technique (numerical integration)
  - Penalty methods
  - Distributional assumptions
  - Convex approximations
  - ...

See Prékopa (1995), Shapiro and Ruszczyński (2003), Shapiro et al. (2009), Branda and Dupačová (2012), Branda (2012, 2013)

Optimization model – individual chance constraints

If we prescribe a small probability level $\varepsilon \in (0,1)$ for violating the loss ratio in each tariff cell, we obtain the following **chance constraints**

$$P\left(L^T_{i_0,i_1,\dots,i_S} \le \hat{LR} \cdot W_{i_0,i_1,\dots,i_S} \cdot Pr_{i_0} \cdot (1+e_{i_1}) \cdot \dots \cdot (1+e_{i_S})\right) \ge 1-\varepsilon,$$

which can be rewritten using the **quantile function** $F^{-1}_{L^T_I}$ of L^T_I as

$$\hat{LR} \cdot W_{i_0,i_1,\dots,i_S} \cdot Pr_{i_0} \cdot (1+e_{i_1}) \cdot \dots \cdot (1+e_{i_S}) \ge F^{-1}_{L^T_{i_0,i_1,\dots,i_S}}(1-\varepsilon).$$

By setting

$$b_I = \ln\left[\frac{F^{-1}_{L^T_I}(1-\varepsilon)}{W_I \cdot \hat{LR}}\right],$$

the formulation (6) can be used.

Optimization model – individual chance constraints

$$\min_u \sum_{I \in \mathcal{I}} w_I e^{u_{i_0}+u_{i_1}+\cdots+u_{i_S}}$$

$$\text{s.t.}$$

$$\begin{aligned} u_{i_0} + u_{i_1} + \cdots + u_{i_S} &\geq b_{i_0,i_1,\ldots,i_S}, \\ u_{i_1} + \cdots + u_{i_S} &\leq \ln(1 + r^{max}), \\ u_{i_1}, \ldots, u_{i_S} &\geq 0, \ (i_0, i_1, \ldots, i_S) \in \mathcal{I}, \end{aligned}$$

with

$$b_I = \ln \left[\frac{F^{-1}_{L_I^T}(1-\varepsilon)}{W_I \cdot \hat{LR}} \right].$$

Optimization model – individual reliability constraints

It can be very difficult to compute the quantiles $F^{-1}_{L_I^T}$, see, e.g., Withers and Nadarajah (2011). We can employ the **one-sided Chebyshev's inequality** based on the mean and variance of the compound distribution:

$$P\left(\frac{L_I^T}{TP_I} \geq \hat{LR}\right) \leq \frac{1}{1 + (\hat{LR} \cdot TP_I - \mu_I^T)^2/(\sigma_I^T)^2} \leq \varepsilon, \qquad (7)$$

for $\hat{LR} \cdot TP_I \geq \mu_I^T$. Chen et al. (2011) showed that **the bound is tight** for all distributions $\mathcal{D}$ with the expected value μ_I^T and the variance $(\sigma_I^T)^2$:

$$\sup_{\mathcal{D}} P\left(L_I^T \geq \hat{LR} \cdot TP_I\right) = \frac{1}{1 + (\hat{LR} \cdot TP_I - \mu_I^T)^2/(\sigma_I^T)^2},$$

for $\hat{LR} \cdot TP_I \geq \mu_I^T$.

Optimization model – individual reliability constraints

The inequality (7) leads to the following constraints, which serve as conservative approximations:

$$\mu_I^T + \sqrt{\frac{1-\varepsilon}{\varepsilon}}\sigma_I^T \leq \hat{LR} \cdot TP_I.$$

Finally, the constraints can be rewritten as **reliability constraints**

$$\mu_I + \sqrt{\frac{1-\varepsilon}{\varepsilon}}\frac{\sigma_I}{\sqrt{W_I}} \leq \hat{LR} \cdot Pr_I. \tag{8}$$

If we set

$$b_I = \ln\left[\left(\mu_I + \sqrt{\frac{1-\varepsilon}{\varepsilon W_I}}\sigma_I\right)/\hat{LR}\right],$$

we can employ the linear programming formulation (6) for rate estimation.

Optimization model – individual reliability constraints

$$\min_u \sum_{I \in \mathcal{I}} w_I e^{u_{i_0}+u_{i_1}+\dots+u_{i_S}}$$

$$\text{s.t.}$$

$$\begin{aligned} u_{i_0}+u_{i_1}+\dots+u_{i_S} &\geq b_{i_0,i_1,\dots,i_S}, \\ u_{i_1}+\dots+u_{i_S} &\leq \ln(1+r^{max}), \\ u_{i_1},\dots,u_{i_S} &\geq 0,\ (i_0,i_1,\dots,i_S) \in \mathcal{I}, \end{aligned}$$

with

$$b_I = \ln\left[\left(\mu_I + \sqrt{\frac{1-\varepsilon}{\varepsilon W_I}}\sigma_I\right)/\hat{LR}\right].$$

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Optimization model – a collective risk constraint

In the collective risk model, a probability is prescribed for ensuring that the total losses over the **whole line of business** (LoB) are covered by the premium with a high probability, i.e.

$$P\left(\sum_{I\in\mathcal{I}} L_I^T \le \sum_{I\in\mathcal{I}} W_I Pr_I\right) \ge 1-\varepsilon.$$

Optimization model – a collective risk constraint

Zaks et al. (2006) proposed the following program for rate estimation, where the **mean square error** is minimized under the reformulated **collective risk constraint** using the Central Limit Theorem:

$$\begin{aligned} &\min_{Pr_I} \sum_{I \in \mathcal{I}} \frac{1}{r_I} \mathbb{E}\left[(L_I^T - W_I Pr_I)^2 \right] \\ &\text{s.t.} \\ &\sum_{I \in \mathcal{I}} W_I Pr_I = \sum_{I \in \mathcal{I}} W_I \mu_I + z_{1-\varepsilon} \sqrt{\sum_{I \in \mathcal{I}} W_I \sigma_I^2}, \end{aligned} \tag{9}$$

where $r_I > 0$ and $z_{1-\varepsilon}$ denotes the quantile of the Normal distribution. Various premium principles can be obtained by the choice of r_I ($r_I = 1$ or $r_I = W_I$ leading to semi-uniform or uniform risk allocations).

Optimization model – a collective risk constraint

According to Zaks et al. (2006), Theorem 1, the program has **a unique solution**

$$\hat{P}r_l = \mu_l + z_{1-\varepsilon} \frac{r_l \sigma}{r W_l},$$

with $r = \sum_{l \in \mathcal{I}} r_l$ and $\sigma^2 = \sum_{l \in \mathcal{I}} W_l \sigma_l^2$. If we want to incorporate the prescribed loss ratio $L\hat{R}$ for the whole LoB into the rates, we can set

$$b_l = \ln \left[\left(\mu_l + z_{1-\varepsilon} \frac{r_l \sigma}{r W_l} \right) / L\hat{R} \right],$$

within the problem (6).

Optimization model – a collective risk constraint

$$\min_{u} \sum_{I \in \mathcal{I}} w_I e^{u_{i_0}+u_{i_1}+\cdots+u_{i_S}}$$

$$\text{s.t.}$$

$$\begin{aligned} u_{i_0} + u_{i_1} + \cdots + u_{i_S} &\geq b_{i_0,i_1,\ldots,i_S}, \\ u_{i_1} + \cdots + u_{i_S} &\leq \ln(1 + r^{max}), \\ u_{i_1}, \ldots, u_{i_S} &\geq 0, \ (i_0, i_1, \ldots, i_S) \in \mathcal{I}, \end{aligned}$$

with

$$b_I = \ln\left[\left(\mu_I + z_{1-\varepsilon}\frac{r_I \sigma}{r W_I}\right) / \hat{LR}\right].$$

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MTPL – segmentation criteria

We consider 60 000 policies with settled claims simulated using characteristics of real MTPL portfolio. The following segmentation variables are used:

- **tariff group**: 5 categories (engine up to 1000, up to 1350, up to 1850, up to 2500, over 2500 ccm),
- **age**: 3 cat. (18-30, 31-65, 66 and more years),
- **region** (reg): 4 cat. (over 500 000, over 50 000, over 5 000, up to 5 000 inhabitants),
- **gender** (gen): 2 cat. (men, women).

Many other available indicators related to a driver (marital status, type of licence), vehicle (engine power, mileage, value), policy (duration, no claim discount). **Real data** for MTPL models: 120 columns and over 8 millions rows.

Software

SAS Enterprise Guide:

- SAS GENMOD procedure (SAS/STAT 9.3) – generalized linear models
- SAS OPTMODEL procedure (SAS/OR 9.3) – nonlinear convex optimization

Parameter estimates

Param.	Level	Overd. Poisson Est.	Overd. Poisson Std.Err.	Overd. Poisson **Exp**	Gamma Est.	Gamma Std.Err.	Gamma **Exp**	Inv. Gaussian Est.	Inv. Gaussian Std.Err.	Inv. Gaussian **Exp**
TG	1	-3.096	0.042	0.045	10.30	0.015	29 778	10.30	0.017	29 765
TG	2	-3.072	0.038	0.046	10.35	0.013	31 357	10.35	0.015	31 380
TG	3	-2.999	0.037	0.050	10.46	0.013	34 913	10.46	0.015	34 928
TG	4	-2.922	0.037	0.054	10.54	0.013	37 801	10.54	0.015	37 814
TG	5	-2.785	0.040	0.062	10.71	0.014	44 666	10.71	0.017	44 679
reg	1	0.579	0.033	1.785	0.21	0.014	1.234	0.21	0.016	1.234
reg	2	0.460	0.031	1.583	0.11	0.013	1.121	0.11	0.014	1.121
reg	3	0.205	0.032	1.228	0.06	0.013	1.059	0.06	0.015	1.058
reg	4	0.000	0.000	1.000	0.00	0.000	1.000	0.00	0.000	1.000
age	1	0.431	0.027	1.539	-	-	-	-	-	-
age	2	0.245	0.024	1.277	-	-	-	-	-	-
age	3	0.000	0.000	1.000	-	-	-	-	-	-
gen	1	-0.177	0.018	0.838	-	-	-	-	-	-
gen	2	0.000	0.000	1.000	-	-	-	-	-	-
Scale		0.647	0.000		13.84	0.273		0.002	0.000	

Employed models

- **GLM** – The approach based on generalized linear models
- **EV model** – Deterministic optimization model with expected value constraints
- **SP model (ind.)** – Stochastic programming problem with individual reliability constraints $\varepsilon = 0.1$
- **SP model (col.)** – Stochastic programming problem with collective risk constraint $\varepsilon = 0.1$

Multiplicative tariff of rates

		GLM		EV model		SP model (ind.)		SP model (col.)	
		G	IG	G	IG	G	IG	G	IG
TG	1	1 880	1 879	3 805	3 801	9 318	14 952	4 400	5 305
TG	2	2 028	2 029	4 104	4 105	9 979	16 319	4 733	5 563
TG	3	2 430	2 431	4 918	4 918	11 704	19 790	5 547	6 296
TG	4	2 840	2 841	5 748	5 747	13 380	23 145	6 376	7 125
TG	5	3 850	3 851	7 792	7 791	17 453	31 718	8 421	9 169
reg	1	**2.203**	**2.201**	.311	.390	.407	.552	.463	.407
reg	2	.775	.776	.057	.121	.177	.264	.226	.195
reg	3	.301	.299	.000	.000	.000	.000	.000	.000
reg	4	.000	.000	.000	.000	.000	.000	.000	.000
age	1	.539	.539	.350	.277	.257	.157	.182	.268
age	2	.277	.277	.121	.060	.105	.031	.015	.107
age	3	.000	.000	.000	.000	.000	.000	.000	.000
gen	1	.000	.000	.000	.000	.000	.000	.000	.000
gen	2	.194	.194	.194	.194	.130	.114	.156	.121

Conclusions (open for discussion)

- **EV model** – good start
- **SP model (ind.)** – appropriate for less segmented portfolios with high exposures of tariff cells
- **SP model (col.)** – appropriate for heavily segmented portfolios with low exposures of tariff cells

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Thank you for your attention. Questions?

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