Hands on Stochastic Models

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May 29, 2013

Pricing

Reinsurance Purchase

Risk Management

Stochastic models

Reserving

Portfolio Management

Capital Modelling

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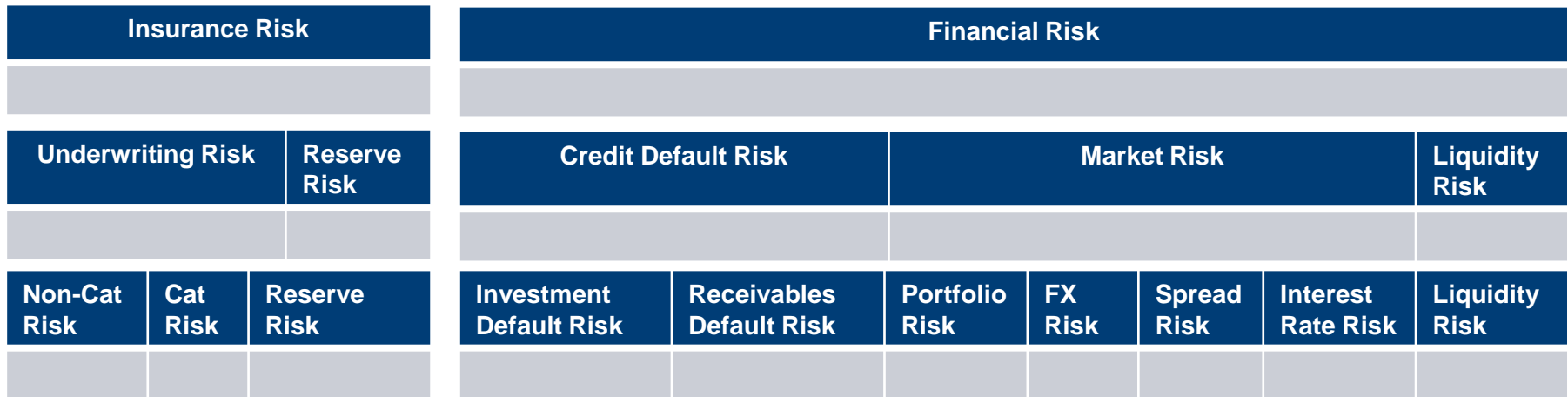
Agenda

- ICA Model
  - Goal
  - Dependencies & Globals
  - Gross UW
  - Catastrophes
  - Reinsurance
  - Reserve Risk
  - Asset And Financial Statements

- 1 Year View
- Dependencies
- Solvency II Financial Statements

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What is ICA Model?

- Capital assessment regime in UK since 2004
  - Value Assets and Liabilities at close Market Value and Risk Based Capital using Internal Models
- Follows UK GAAP
  - Based on earnings - in line with business thinking
  - To ultimate
  - Sets capital at 99,5th Percentile (equivalent to BBB)
  - Used for rating agencies capital settings
- Various models across the market
  - UW year vs AY based
  - Starting with zero balance
  - Right level of simplifications and assumptions – unable to model reality
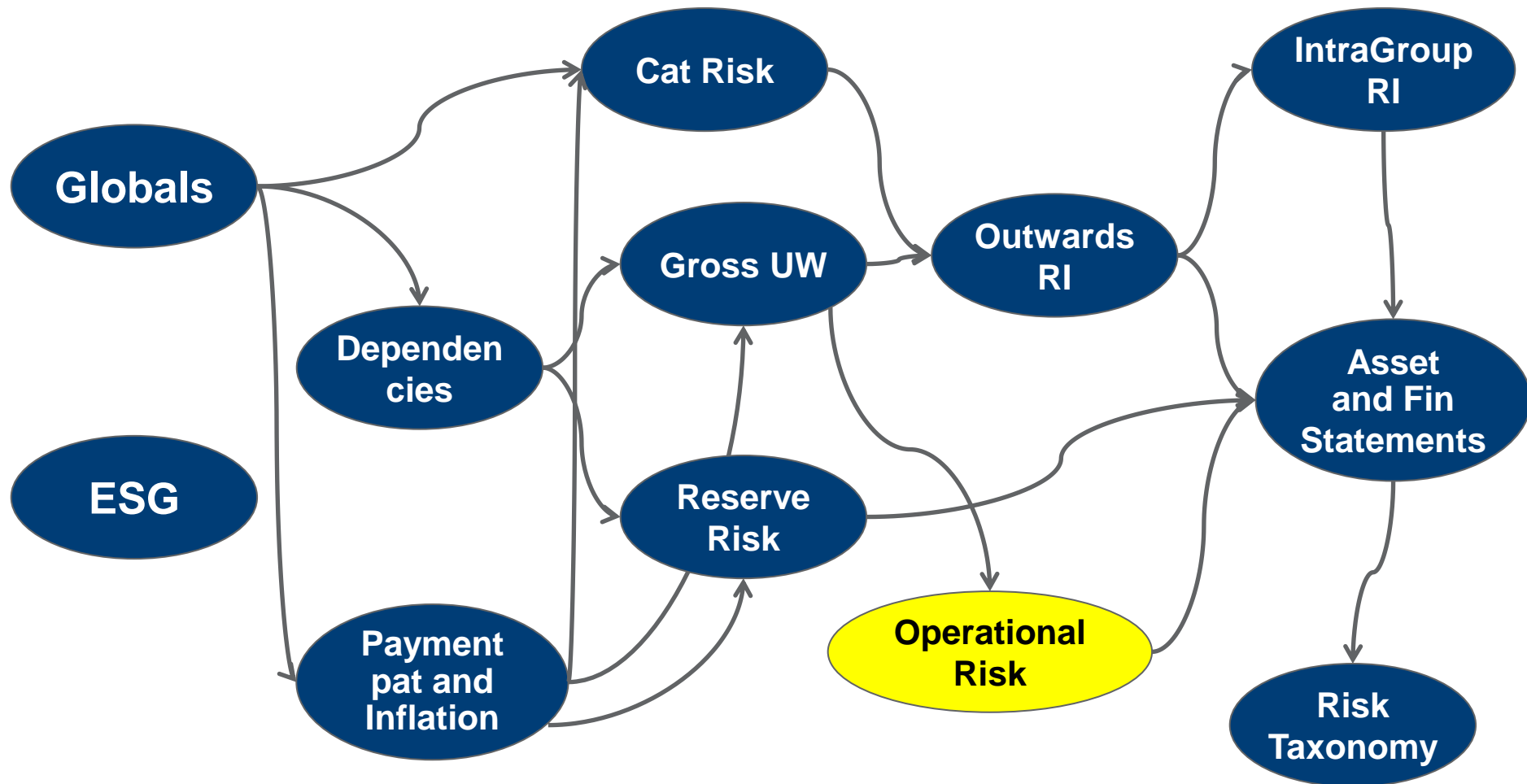- Transition between ICA to SII

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Goal

- **Financial Statements**
  - P&L (by YoA, AY, RunOff)
  - Balance sheet (Opening position – calculation consistency)
- **Risk Taxonomy**

All Risk		+	Operational Risk

Insurance Risk		Financial Risk				

Underwriting Risk	Reserve Risk		Credit Default Risk		Market Risk					Liquidity Risk

Non-Cat Risk	Cat Risk	Reserve Risk		Investment Default Risk	Receivables Default Risk	Portfolio Risk	FX Risk	Spread Risk	Interest Rate Risk	Liquidity Risk

- **Other output (Reinsurance, LoB, Cat – Gross/Net)**   Add 1

Model Map

Globals & Dependencies

- ## Globals

Frequency Dist	Severity Dist	Currency	RI Lags	RI_Programmes Types	Trigger Mechanism	Periods	Classes Modelled	Decision	Priority
Scheduled	LogNormal	USD	0	QS	LOD	1	Class A	Yes	1
Binomial	Exponential	GBP	0.25	XoL	RAD	2	Class B	No	2
Poisson	General Pareto		0.5						3
Negative Binom	Gamma		0.75						
	Weibull		1						
	Logistic								

- ## Dependencies

Correlation	Class A Reserves	Class A UW - Attritional	Class A UW - Large Frq
Class A Reserves	1	0.433012702	0.433012702
Class A UW - Attritional	0.433	1	0.25
Class A UW - Large Frq	0.433	0.25	1

Add 1

Add 2

Add 3

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Gross UW - Inputs

UW Year	2013			
Class of Business - Name	Class A	Class B		
Dominant Currency	USD	GBP		
UPR	69,303,637	33,110,835		
Written Premium	108,304,093	48,554,541		

Earning Pattern	Class A		Class B	
Earning Pattern	37%	63%	41%	59%

Non-Cat Parameters	Class A		Class B	
	Attritional Losses	Large Losses	Attritional Losses	Large Losses
Frequency Distribution	N/A - Aggregate distribution	Poisson	N/A - Aggregate distribution	Poisson
Freq param - 1	N/A - Aggregate distribution	3.41	N/A - Aggregate distribution	7.09
Freq param - 2	N/A - Aggregate distribution		N/A - Aggregate distribution	
Severity Distribution	Logistic	LogNormal	Weibull	LogNormal
Sev Param - 1	0.40	14.42	2.21	14.29
Sev Param - 2	0.07	0.74	0.50	0.40
Sev Param - 3	0	-	0	-
LowerBound	-	1,195,676	-	750,000
UpperBound (Cap)	-	15,942,349	-	25,000,000
Translation	-	-	-	-

+ Expenses (Fixed, Variable, Acquisition)

Gross UW - methodology

- Modelled on AY basis (Business earned over 2 years)
    - Gross Gross Premium
    - Underwriting x Accident Date Pattern
    - Expenses
        - Fixed
        - Variable (Normal Distribution), % of Gross Premium
        - Acquisition
    - Loss Distributions

- 2nd Year is scaled by exposure
    - Attritional Losses
    - Large Loss Frequency

$$\frac{AY2\ Earned\ Premium}{AY1\ Earned\ Premium}$$

- UW Year Split
    - Attritonal & Expenses – based on Premium
    - Large – assigned to UW Years

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Gross UW - model

Premium Earnings	Class A		Class B	
	AY1	AY2	AY1	AY2
Earned Premium	109,413,949	68,193,780	52,924,378	28,740,997

1st AY Split to UWYears	Class A		Class B	
	2012 UW Year	2013 UW Year	2012 UW Year	2013 UW Year
1 AY Split Pct	63%	37%	63%	37%

Expenses	Class A		Class B	
	AY1	AY2	AY1	AY2
Operating Expenses	9,738,967	6,231,951	4,207,042	2,868,911
Operating Variable Expenses	-	-	-	-
Acquisition Expenses	13,078,770	22,235,946	4,306,870	6,247,431
Opening DAC	22,012,758	-	7,768,516	-

Attritional Losses		
	Class A	
	AY1	AY2
Attritional Losses - Aggregate	28,799,389	35,383,345

Large Losses		
	Class A	
	AY1	AY2
Large Losses - Frequency	4	3
Large Loss - Severity - Loss No.	AY1	AY2
1	1,571,336	1,503,007
2	2,287,673	1,910,101
Large Loss UWYear Indicator	AY1	AY2
1	-	1
2	-	1

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Cat Risk – Input & Methodology

- Input from RMS/AIR

EVENT ID	RATE	PERSPVALUE	STDEVI	STDEVC	EXPVALUE
151076	0,00897	15629,0	24387,9	48775,8	97551,6
158525	0,00913	50150,5	19561,2	39122,5	78245,0
171773	0,00162	11337,0	21910,0	43820,1	87640,1
114034	0,00999	10618,4	9328,3	18656,6	37313,2
185157	0,00826	589,0	243,7	487,4	974,7
...	...	...	...	...	...
167638	0,00473	5443,0	13011,5	26022,9	52045,9

- Frequency - Poisson
- Secondary uncertainty - Damage Ratio (Beta distribution)

<u>Add 1</u>

$$\alpha = \left(\frac{EX(1-EX)}{VarX} - 1 \right) EX \qquad EX = PERSPVALUE$$

$$\beta = \left(\frac{EX(1-EX)}{VarX} - 1 \right) (1-EX) \qquad VarX = \left(\frac{STDEVI + STDEVC}{2} \right)^2$$

- Loss = EXPVALUE*Damage Ratio

Cat Risk - Model

Cat Losses				
	Class A		Class B	
	AY1	AY2	AY1	AY2
Cat Losses - Frequency	0	0	0	0
Cat Loss - EventID	AY1	AY2	AY1	AY2
1				
2				

Cat Loss - Severity	AY1	AY2	AY1	AY2
1				
2				

Reinsurance Inputs

Programme Name	XoL Programme 1	XoL Programme 2	QS Programme 3
Programme Type	XoL	XoL	QS
Programme Details	XoL Programme 1	XoL Programme 2	QS Programme 3
Innurance priority	1	1	1
Currency	USD	USD	USD
Trigger machanism	RAD	RAD	RAD
Cession	50%	50%	50%
Limit	4,000,000	4,000,000	1.00E+99
Attachment	40,000,000	20,000,000	0
RI Premium (pct of subject premium)	10.8%	8.3%	50%
No. Reinstatements	2	2	0
Reinstatement Premium (pct of RI Premium)	100%	100%	0
Period Covered	1	1	1
Commission			37%
RI Share UPR			66,761,947

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Reinsurance - methodology

- QS, XoL, Cat XoL, SL, ILW, CWIL
  - LOD vs RAD
  - Reinstatements
  - Ceding/Profit Commissions vs Adjustment Premium
  - Premium based on subject premium, Rate on Line
  - Cession
  - Inurance priority
  - Slip Rates
- Modelling simplifications
  - Mid Year Inceptions vs Annual Inceptions
  - Paid by Paid basis vs payment lag
  - First come First serve vs. Contribution to total recoveries

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Reinsurance - Model

RI Premium	XoL Programme 1	XoL Programme 2	QS Programme 3
Subject Premium	108,304,093	77,063,131	185,367,225
Earning	37%	41%	38%
Base RI Premium	11,642,690	6,357,708	92,683,612

Recoveries - UnScaled (unScaling only For XoL purposes)	XoL Programme 1		XoL Programme 2		QS Programme 3	
	AY1	AY2	AY1	AY2	AY1	AY2
Attritional - Class A					5,278,817	6,485,631
Attritional - Class B					4,846,429	2,243,995
Large Losses - Class A						
1	-	-			-	751,503
2	-	-			-	955,051

Recoveries - Scaled (Scaling only For XoL purposes)	XoL Programme 1		XoL Programme 2		QS Programme 3	
	AY1	AY2	AY1	AY2	AY1	AY2
Attritional - Class A					5,278,817	6,485,631
Attritional - Class B					4,846,429	2,243,995
Large Losses - Class A						
1					-	751,503
2					-	955,051

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RI Default – Inputs

Reinsurer Placement	XoL Programme 1	XoL Programme 2	QS Programme 3
Reinsurer A	0%	0%	0%
Reinsurer B	50%	50%	80%
Reinsurer C	50%	50%	20%

S&P Ratings	Reinsurer Rating
Reinsurer A	AAA
Reinsurer B	AA
Reinsurer C	B

Rating Table	Default Probability	Post Default Recovery Rate Mean	Post Default Recovery Rate STD
AAA	0.14%	70%	10%
AA	0.15%	70%	10%
A	0.34%	70%	10%
BBB	1.20%	70%	10%
BB	5.17%	70%	10%
B	15.15%	70%	10%

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RI Default – Methodology & Model

- **Rating Transitions**
  - Transition matrix is not industry specific (Financial market – prudent)
- **Straight to ultimate**
  - Using 3 year default rate (based on Liability duration)
- **Default modelled on for each cashflow year**
  - Defaulted Reinsurers are replaced with reinsurers in same rating bucket

- **Model**

Reinsurer Default - Calculation	Default (Yes/No)	Recovery Rate	Loss Rate
Reinsurer A	0	0	0
Reinsurer B	0	0	0
Reinsurer C	1	0.627626515	0.372373485

Default by Programme - calculations	XoL Programme 1	XoL Programme 2	QS Programme 3
Default Pct	0.186186743	0.186186743	0.074474697

- Default – Binomial distribution
- Loss given default – Beta distribution

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Reserve Risk – Inputs

Opening Reserves	Class A		Class B	
	Opening Gross Reserve	Opening Net Reserve	Opening Gross Reserve	Opening Net Reserve
Currency	USD	USD	GBP	GBP
2012	15,525,367	6,598,281	12,000,109	5,063,495
2011	26,058,723	12,896,281	21,704,697	10,891,149
2010	25,203,058	24,437,618	21,876,299	20,705,261
2009	19,712,020	18,998,461	19,800,540	18,365,755
2008	16,808,052	16,808,050	18,217,367	15,109,432
2007 and prior	39,858,120	37,491,159	24,956,597	20,247,246

Reserve Volatility Parameters	Class A		Class B	
(LogNormal Distribution)	Opening Gross Reserve	Opening Net Reserve	Opening Gross Reserve	Opening Net Reserve
EX - (Mean)	1.00	1.00	1.00	1.00
Std - (Standard Deviation)	0.03	0.03	0.03	0.03
mu	- 0.01	-0.01	-0.01	-0.01
sigma	0.16	0.16	0.16	0.16

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Reserve Risk Methodology & Model

- Reserve uncertainity modelled using LogNormal distribution
    - Straight to ultimate; aggregate level
    - RI share of Reserve modelled using Gross to Net ratio
        - % vary by percentile to allow to XoL Recoveries and exhaustion of covers
    - RI Default – modelled on total exposure level

Reserve Risk				
	Class A		Class B	
	USD	GBP	USD	GBP
Reserve Volatility	0.84		0.95	
Gross Reserve	120,011,412	-	-	112,694,954
Net Reserve	98,270,432	-	-	85,914,395

Total Reserves - Ultimate	USD	GBP
Gross	120,011,412	112,694,954
Net Reserve	98,270,432	85,914,395

Investment Asset Methodology

- Modelled over 1 year

- High level asset type split

    - Cash, Equity, Corporate Bonds, Government Bonds

    - Split into buckets (currency, rating, maturity, country)

    - Calculation consistency at T0 and T1

- Asset default

    - Black swans events

    - Difficult to estimate diversification effect for some asset buckets

    - No default risk for government bonds denominated in country currency (except Euro zone)

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Liquidity, Receivables Default & Operational Risk

- Not part of the simplified model

- Liquidity Risk
  - Running out of cash
  - Borrow vs selling other assets
- Receivables Default Risk
  - Modelled same as RI default Risk
  - Broker balances
- Operational Risk
  - Frequency severity approach
  - Aprox. 10% of total capital (BBB)

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Profit & Loss Statement

- Accounted in line with UK GAAP
- Opening position calculated by model
  - Calibration and consistency check
- Balance sheet movement explained by P&L
- Mean P&L in line with Business Plan

Ultimate P&L		
Investment Income - RFR		7,961,928
Investment Income - Spread		4,100,027
Capital Gain& Losses	-	2,136,239
FX Gain & Losses	-	5,628,566
Investment Expenses and Charges		-
Net Earned Premium		222,460,235
Expenses (Acq + Opt)		79,317,004
Loss and Loss Expenses		116,153,990
P&L Result		31,286,392
Check	-	0

Add 1

Add 2

Add 3

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Risk Taxonomy

- ## Consistent with P&L

Single simulation - USD					
Retained Earnings	- 31,286,392				
Insurance Risk	- 28,834,720	Underwriting Risk	- 2,784,026	Non Catastrophe Risk	- 2,784,026
				Catastrophe Risk	-
		Reserve Run Off Risk	- 26,050,694	Reserve Run Off Risk	- 26,050,694
Financial Risk	- 2,451,672	Credit Default Risk	1,845,478	Investment Default Risk	-
				Reinsurer Default Risk	1,845,478
				Receivables Default Risk	-
		Market Risk	- 4,297,150	Portfolio Risk	-
				FX Risk	5,628,566
				Spread Risk	- 4,100,027
				Interest Rate Risk	- 5,825,689
		Liquidity Risk	-	Liquidity Risk	-
Other Risk	-	Operational Risk	-	People / Process / Systems Risk	
				External Event Risk	
				Group Risk	
		Strategic Risk	-	Reputational Risk	

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Portfolio management & Future of GI Models

- How to allocate capital down to individual contract?
- How to capture accumulations within LoB?
  - New business analysis
  - Fac/whole account reinsurance
- Model point approach
  - Policy by policy
  - Capturing trends
  - Dependencies on policy level
  - Currently for small classes
  - Allow capturing policy specifics
- Link Pricing with Capital modelling and Reserving
  - Parameterisation

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Conclusion & Questions

- Building a model
    - Easy/Ability to parameterise
    - Easy to understand
    - Modelling proportionality
    - Runtime Runtime Runtime
        - Don't underestimate and also overestimate level of dependencies
    - Use test
- It's only a model

Questions?

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Dependencies – Matrix Adjustment

- ## Method 1

  $$M' = \frac{1}{1 - \lambda_{\min}}(M - I\lambda_{\min})$$

  - Formula

$$M = \begin{pmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.3 \\ 0.7 & 0.3 & 1 \end{pmatrix}$$

$$\left\{ 2.2967 \quad 0.710625 \quad -0.00735 \right\}$$

$$M' = \begin{pmatrix} 1 & 0{,}8934 & 0{,}6949 \\ 0{,}8934 & 1 & 0{,}2978 \\ 0{,}6949 & 0{,}2978 & 1 \end{pmatrix}$$

$$\left\{ 2{,}287 \quad 0{,}713 \quad 0 \right\}$$

- ## Method 2
  - Calculate eigenvalues and right hand side eigenvectors of M
  - Set all negative eigenvalues to 0
  - Set the length of the eigenvector to its associated eigenvalue
  - Arrange the eigenvectors as the columns of the matrix C
  - C' results from C by normalising the row vectors of C to unit length
  - Calculate M'=C'C'^{T}

$$C = \begin{pmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.3 \\ 0.7 & 0.3 & 1 \end{pmatrix}$$
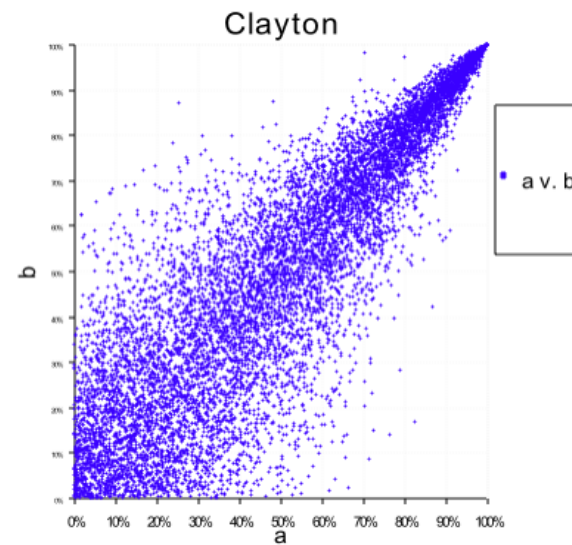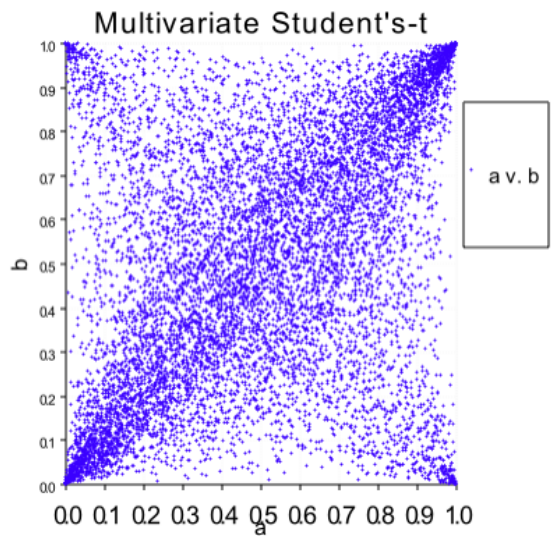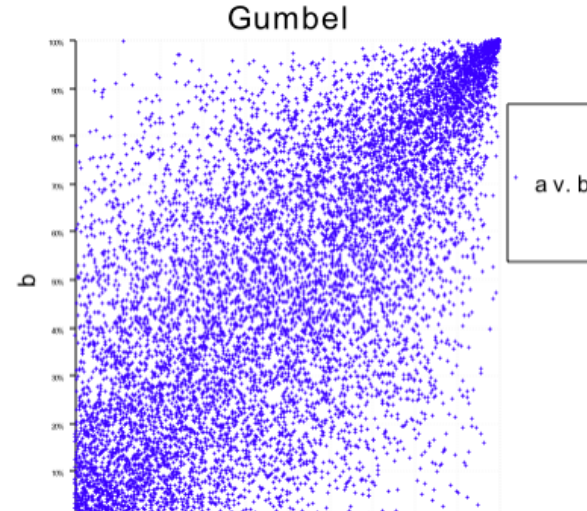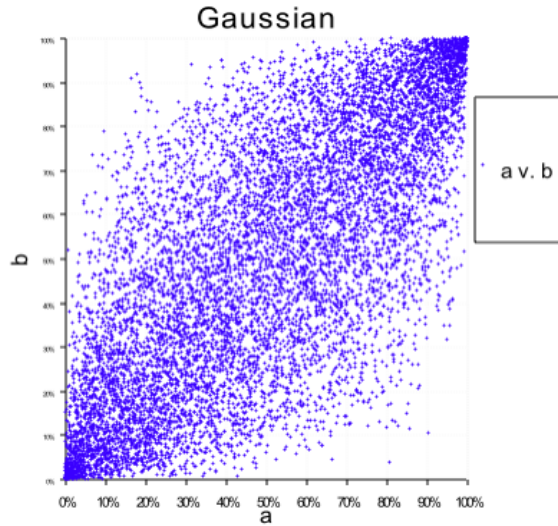
$$\left\{ 2.2967 \quad 0.710625 \quad -0.00735 \right\}$$

$$C = \begin{pmatrix} 1 & 0.89458 & 0.69662 \\ 0.89458 & 1 & 0.30254 \\ 0.69662 & 0.30254 & 1 \end{pmatrix}$$
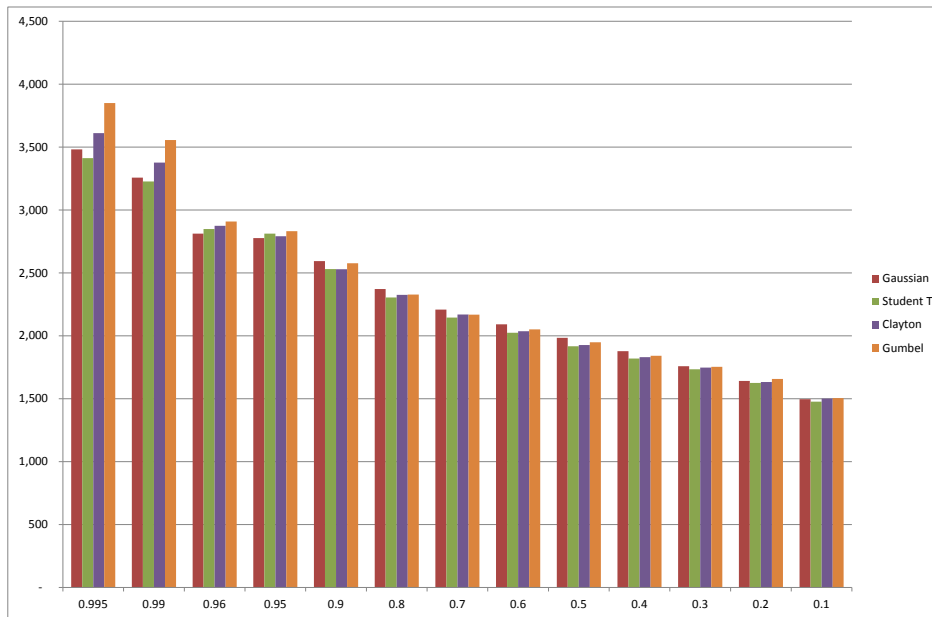
$$\left\{ 2.2967 \quad 0.710625 \quad 0 \right\}$$

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Dependencies Tail vs Rank

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Dependencies - Example

	Mean	Std
Class A	1000	200
Class B	1000	300

Target Correlation	**0.5**
Degrees of freedom	**3**

Percentile	Gaussian	Student T	Clayton	Gumbel
0.995	3,481	3,412	3,611	3,849
0.99	3,257	3,226	3,376	3,556
0.96	2,812	2,848	2,874	2,909
0.95	2,776	2,811	2,791	2,831
0.9	2,593	2,530	2,528	2,577
0.8	2,371	2,304	2,326	2,328
0.7	2,208	2,145	2,169	2,168
0.6	2,090	2,023	2,037	2,051
0.5	1,984	1,917	1,926	1,949
0.4	1,878	1,819	1,830	1,841
0.3	1,759	1,734	1,747	1,753
0.2	1,641	1,625	1,633	1,656
0.1	1,496	1,477	1,501	1,504

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Cat - non-RMS/AIR parameterisation

Name of Peril	PML 1	RP 1	PML 2	RP 2	Max Loss
Flood Australia	5 000 000	25	15 000 000	250	22 000 000

- Frequency - Poisson $\qquad \lambda = \dfrac{1}{RP1}$

- Severity - Pareto

$$F(x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha}; \alpha > 0, \beta > 0, x > \beta$$

$$\beta = PML1$$

$$\alpha = \ln\left(\frac{-\ln\left(1 - \dfrac{1}{RP2}\right)}{\lambda}\right) \frac{1}{\ln\left(\dfrac{PML2}{PML1}\right)} = 2.094$$

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1 Year View

- ## Gross Loss Emergence
  - Complete Re-Reserving in model - not recommended
  - Perfect view on ultimate - for some risks
  - Factor approach (weights between mean and true ultimate) - preferred approach
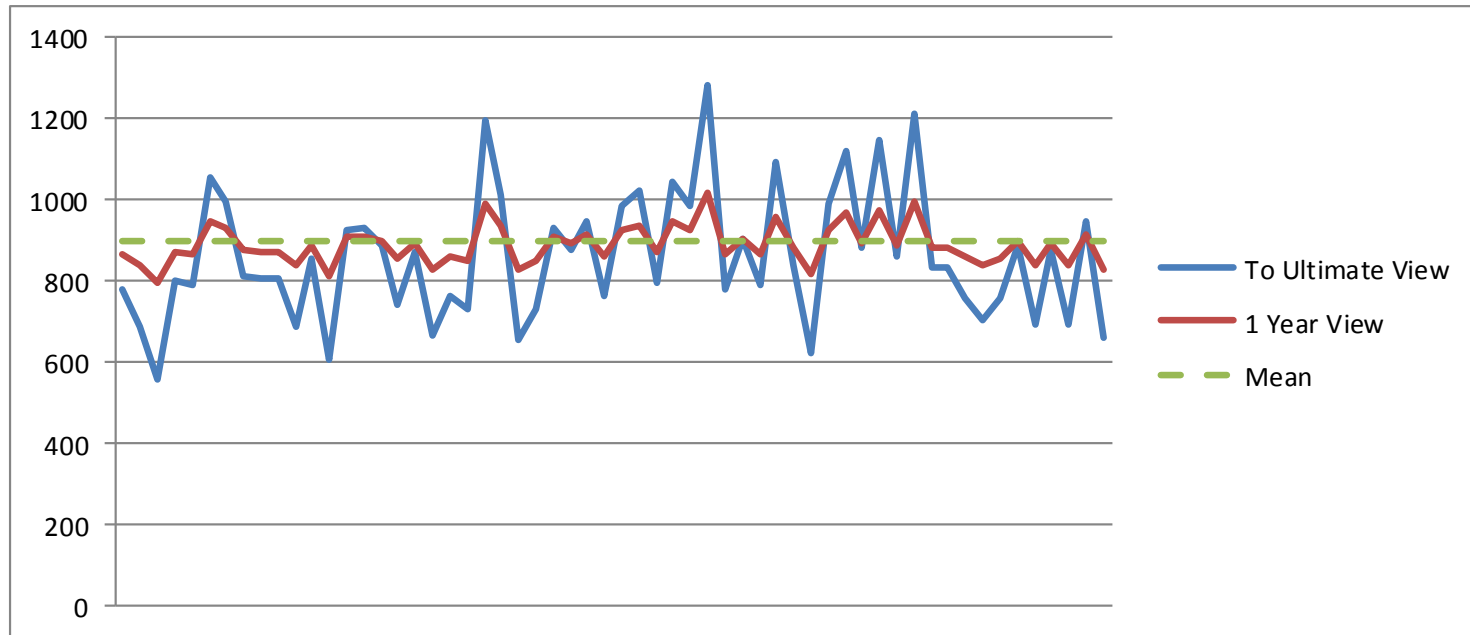   - Simple and easy to validate

$$EX + \alpha(X - EX) = (1 - \alpha)EX + \alpha X$$

- ## RI Recoveries Emergence
  - Calculate recoveries for 1 year gross views
    - Same proportion between emergence and ultimate for all losses
  - Apply the gross loss emergence factors to ultimate RI recoveries
    - Limitations (ART, non-proportional contracts)

- ## E[1 Year View]=E[To Ultimate]

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1 Year View – Liability class – 30% Emergence

Class of business	20% CoV
Premium	1000
Loss Ratio	90%
Capital To ultimate	479
Risk Free Rate	2%
Cost of Capital	6%
Capital To 1YR	74
Increase in Risk Margin	28
Total Capital on 1YR basis	102

Solvency II P&L

Economic Profit and Loss Statement	
PAID PREMIUMS	
Gross Received Premium	
Change in Gross Premium Provision	
Paid RI Premium	
Change in RI Share of Premium Provision	
Total Net Paid Premium	
PAID EXPENSES	
Gross Acquisition Cost Paid	
Ceded Acquisition Cost Received	
Operating Costs Paid	
Inwards RI Commissions Paid	
Outwards RI Commissions Paid	
TOTAL NET PAID EXPENSES	

LOSSES	
Gross Losses Incurred	
RI Recoveries Incurred	
Change in Risk Margin	
TOTAL NET LOSSES	
UNDERWRITING PROFIT	
OTHER REVENUE	
Net Investment Income	
Gains and Losses (Inv, FX)	
TOTAL OTHER REVENUE	
PRE TAX PROFIT	

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Risk Taxonomy – Example

- P&L Type output x Surplus Type output

	Premium	Expenses	LR - Mean	LR - SD
UW Risk	1000	100	80%	10%
	Opening Gross Reserve		UW x Reserve Correl	
Reserve Risk	1000		10%	

	P&L Approach	Surplus Approach
UW Risk	185.44	285.44
Reserve Risk	612.90	612.90
Insurance Risk	610.15	710.15
	-	100.00
Capital	610.15	610.15
Standard Formula Correl	-17%	13%

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