May 13th, 2022

Modelling of MTPL Risks in Reinsurance

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Modelling of MTPL Risks in Reinsurance

Introduction

Motor Third-Party Liability Insurance

- **MTPL (Motor Third-Party Liability Insurance) is covering all bodily injuries, property damage and pure financial losses to others caused by use of a vehicle.**
- **This type of insurance is compulsory in nearly all countries and is highly regulated.**
- **Coverage is varying by country.**
  - e.g., in the UK, the coverage applies to the driver personally (even if driving somebody else's car), but in the Czech Republic or Germany it is linked to the vehicle.
- **Generally, each country defines minimum statutory limits of coverage.**
  - but e.g., in France and in the UK the coverage is unlimited by law.

Motor Third-Party Liability Insurance

- **MTPL insurance covers claims occurring in the country where insurance coverage was bought (Domestic coverage) and abroad (Green Card coverage).**

- **Scope of Green Card coverage is defined by the current legislation.**

- **EEA plus additional European countries are covered (except for Kosovo).**

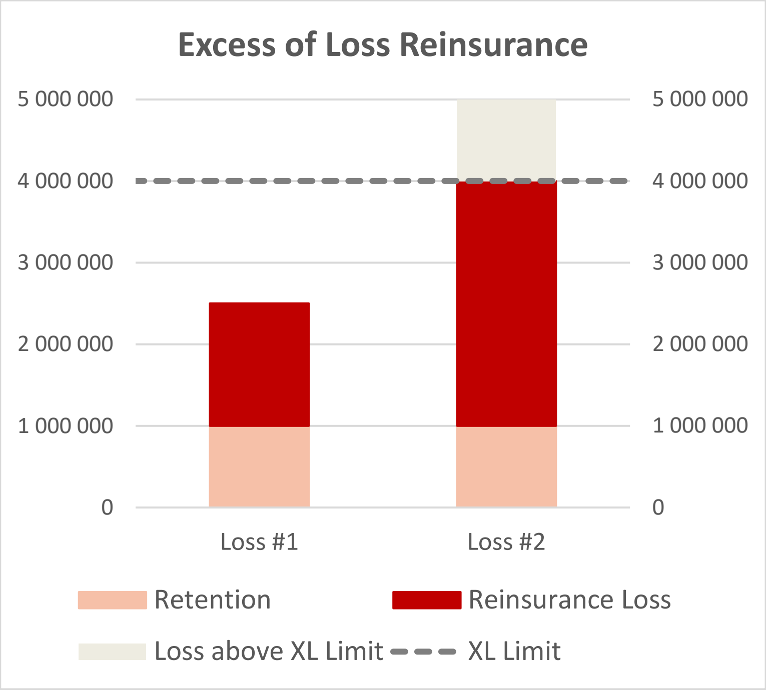
Reinsurance of MTPL business

- **Excess of Loss (XL) reinsurance contract protects the insurance company from extreme-scale events (e.g., Green Card claims in Western Europe).**
  - Smoothens the result of the client's MTPL book.

- **Quota Share (QS) reinsurance contract helps to increase underwriting capacity of the insurance company.**
  - Decreases capital required to write more business.

Excess of Loss (XL) Reinsurance Contract

- **Covers part of the claim in excess of a specific amount (deductible/retention) up to a defined limit.**
- **Can be layered.**
- **For MTPL Green Card claims, the coverage is generally unlimited (due to the possibility of unlimited losses in Western Europe).**



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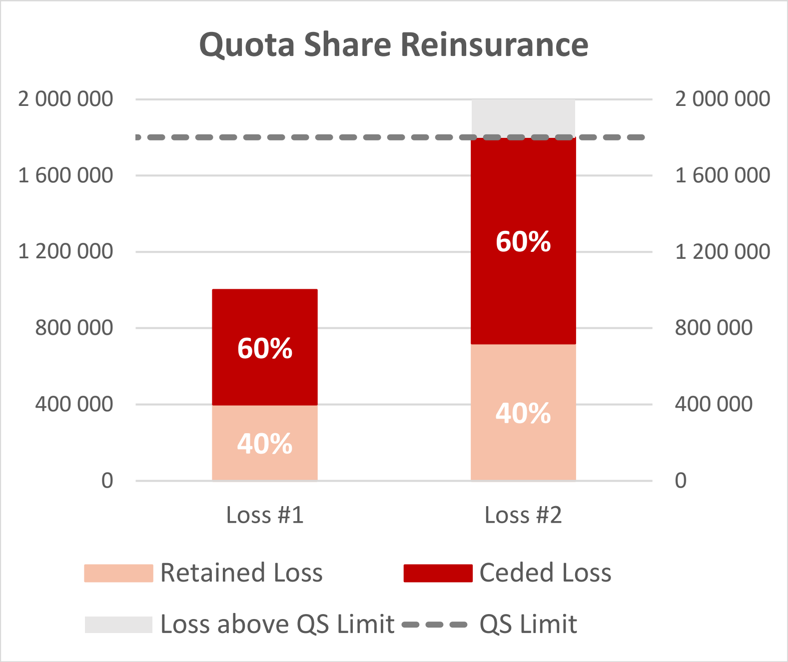
Quota Share (QS) Reinsurance Contract

- **Covers specific percentage of each and every insurance claim.**
- **In MTPL reinsurance, QS could be limited to the retention of an XL contract.**
- **Insurance company could benefit from the increase in underwriting capacity or market expertise of the reinsurer.**



Information from Insurance Company

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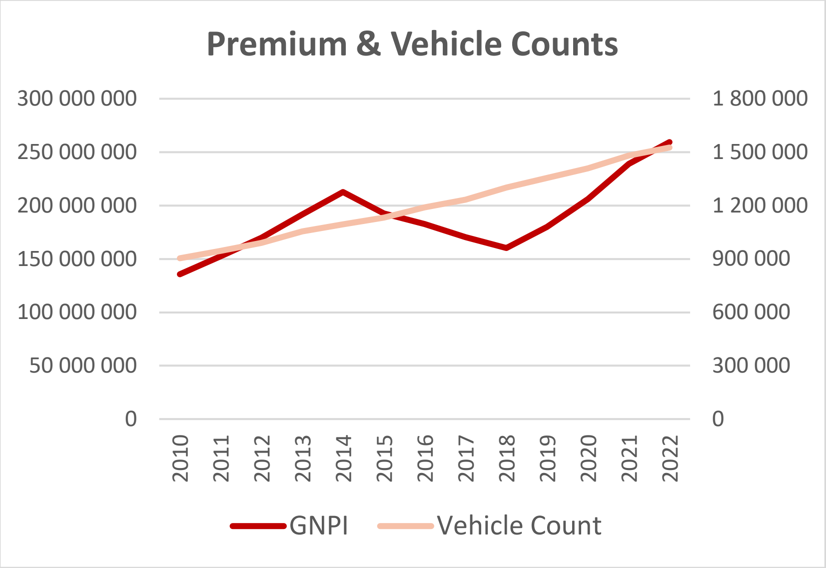
Client data

Information from Insurance Company

- **Historical premium development and vehicle counts**

Year	GNPI	Vehicle Count
2010	135 678 234	904 522
2011	152 480 506	945 476
2012	169 777 056	990 289
2013	191 611 408	1 055 621
2014	212 707 202	1 094 113
2015	192 576 604	1 132 930
2016	182 794 639	1 190 432
2017	170 521 570	1 232 988
2018	160 198 249	1 301 885
2019	179 893 158	1 354 501
2020	206 019 850	1 408 387
2021	239 098 339	1 480 197
2022	259 471 217	1 526 301



Information from Insurance Company

- **It is important to consider the development of the average premium.**
- **Market knowledge is necessary to make conclusions based on data.**
- **Is the estimate for the next year reliable?**

Year	Average Premium	Change %
2010	150	
2011	161	7,5%
2012	171	6,3%
2013	182	5,9%
2014	194	7,1%
2015	170	-12,6%
2016	154	-9,7%
2017	138	-9,9%
2018	123	-11,0%
2019	133	7,9%
2020	146	10,1%
2021	162	10,4%
2022	170	5,2%

Apparently, competition was present in the market

Can we believe it, or the market will return to competition??

Client data

Information from Insurance Company

- **Portfolio composition:**
  - personal cars, trucks, buses etc.
- **It is important to track the percentage of heavy vehicles (trucks, lorries, buses).**
- **It is recommended to compare current risk profile to previous year(s) to see how the portfolio is developing.**

Type of vehicle	Count
Motorcycles	52 185
Passenger Cars	1 231 080
Vans	10 387
Lorries (Weight 3,500kg -12,000kg)	96 509
Trucks (Weight above 12,000kg)	55 078
Trailers	17 342
Buses/Coaches	295
Trolley Bus / Tramway	0
Ambulances / Rescue vehicles	11
Other vehicles	17 310
TOTAL	**1 480 197**

Information from Insurance Company

- **Triangulation of large losses**
  - reporting threshold for the losses is usually not higher than 50% of reinsurance deductible.

- **Country of claim occurrence per each claim (split for Domestic / Green Card claims)**

Claims in excess of 50% of the retention (1 000 000 EUR)

UY	Date of Loss	Currency	Country of accident	Type of vehicle	Schedule	@ 31.12.2000	@ 31.12.2001	@ 31.12.2002	@ 31.12.2003	@ 31.12.2004	@ 31.12.2005	@ 31.12.2006	@ 31.12.2007	@ 31.12.2008
2000	28.10.2000	EUR	Czech Republic	Passenger Car	Claims Paid	182 096	210 034	395 467	509 123	1 234 891	1 432 968	2 042 421	2 042 421	2 042 421
					Reserve RBNS	-	405 891	1 432 067	1 542 228	793 452	609 453	-	-	-
					Total -Claims Incurred	182 096	615 925	1 827 534	2 051 351	2 028 343	2 042 421	2 042 421	2 042 421	2 042 421
2000	17.07.2000	EUR	Germany	Truck	Claims Paid	-	-	304 567	354 567	403 257	1 543 981	2 345 781	2 567 123	5 621 456
					Reserve RBNS	5 983 157	6 591 347	6 293 173	7 894 216	7 845 526	6 704 802	8 332 781	8 146 333	5 181 801
					Total -Claims Incurred	5 983 157	6 591 347	6 597 740	8 248 783	8 248 783	8 248 783	10 678 562	10 713 456	10 803 257

Information from Insurance Company

- **Information on MTPL limits of the country where the client is domiciled**

Current Limits:

	Material Damage		Bodily Injury	
Minimum Domestic Statutory Limit	35 000 000	per event	35 000 000	per person
Minimum Limit offered by the Cedant	35 000 000	per event	35 000 000	per person

	Material Damage		Bodily Injury	
Maximum Usual Limit offered by the Cedant	150 000 000	per event	250 000 000	per person

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Modelling of MTPL Risks in Reinsurance

Data Preparation

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Major Steps of Data Preparation

1. **Inflation of premium**
   - Usually inflated using historical CPI to reflect overall trend in consumer prices.
2. **Preparation of losses**
   - Converting Accident Year triangle to Development Year triangle, potential calculation of cumulative paid status.
3. **Separation of losses into Green Card and Domestic by country of loss**
4. **Inflation of losses**
   - If possible, Wage index should be considered to reflect wage trends which influence claims involving loss of income.
   - Superimposed inflation should be considered.
   - Domestic and Green Card losses are inflated separately.
   - For Green Card losses blended index involving European countries could be considered.
5. **Potential consideration of IBNER factors**

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Inflation of Losses (1)

- **Generally, bodily injury forms the major part of a large MTPL claim covered by an XL treaty.**

- **In case of a serious injury, the damaged party may lose income for a long(er) period of time.**

  - Wage inflation index should be considered to transform past losses to current wage values.

- **Discount rates can have a serious impact on RBNS, especially in case of uncertainty around future rate developments.**

  - E.g., negative Ogden rate in the UK in 2017 had a huge impact on loss reserves.

Inflation of Losses (2)

Let us denote the following:

$I_{x,t}$ chain inflation index between years x and t, where x is the year of claim occurrence and t is the treaty year.

$C_{x,z}^{P}$ status of paid claim value in the development year z if the claim occurred in year x.

$C_{x,z}^{R}$ status of claim reserve in the development year z if the claim occurred in year x.

Inflation of Losses (3)

Inflation of Paid claims

- we inflate incremental paid values from development year z:

AY/DY	1	2	…	z
x	$C^{P}_{x,1} \cdot I_{x,t}$	$C^{P\,INFL}_{x,1} + (C^{P}_{x,2} - C^{P}_{x,1}) \cdot I_{x+1,t+1}$	…	$C^{P\,INFL}_{x,z-1} + (C^{P}_{x,z} - C^{P}_{x,z-1}) \cdot I_{x+z-1,t+z-1}$

$C^{P\,INFL}_{x,1}$

Projections of wage index for years t+1, $t+2$, … should be used!

Inflation of Reserved claims

- we inflate claim reserve status from development year z:

AY/DY	1	2	…	z
x	$C^{R}_{x,1} \cdot I_{x,t}$	$C^{R}_{x,2} \cdot I_{x+1,t+1}$	…	$C^{R}_{x,z} \cdot I_{x+z-1,t+z-1}$

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Superimposed inflation

- **Superimposed inflation can be defined as 'the tendency for benefits for a given injury to increase over time at a faster rate than a suitable standard measure of inflation'.**

- **The drivers of superimposed inflation include:**

  - Rapid price increases related to medical costs, medical equipment, nursing;

  - An increased level of legal involvement (i.e., a higher proportion of legally; represented claimants and hence higher awards);

  - Other social, behavioural and economic factors.

- **Market data analysis could help to estimate superimposed inflation.**

IBNER Factors

- **IBNER = Incurred But Not <u>Enough</u> Reported claims:**
  - Reflects the development of known claims.
- **IBNER together with IBNYR (Incurred But Not Yet Reported claims or pure IBNR) represent overall IBNR reserves.**
- **IBNER development factors should be considered if the insurance company is constantly under-/over-reserving.**
- **Standard reserving methods (Chain Ladder, Mean Link Ratios) should be used to estimate loss development factors.**

Modelling of MTPL Risks in Reinsurance

Threshold Setting

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How to Set a Threshold for Claims Fitting?

<u>Threshold value is not appropriate if:</u>

- **lower than the highest value of the inflated reporting threshold(s) of the insurance company;**
- **higher than the retention of an XL treaty;**
- **in-between two claims of a similar size;**
- **too low;**
  - in case of too many observations for severity fitting, the tail of the distribution might not be estimated properly because of large number of claims concentrated at the bottom part of the curve.
- **too high;**
  - in case of not enough observations for curve fitting, the distribution parameter estimate might not be reliable.

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Extreme Value Theory – GPD

Generalized Pareto Distribution (GPD) plays an essential role in the analysis of the events which exceed a certain high threshold.

The distribution function of the GPD is defined as follows:

$$W_{\gamma,\mu,\sigma}(x) = 1 - \left(1 + \frac{\gamma(x-\mu)}{\sigma}\right)^{-\frac{1}{\gamma}},$$

where $\mu \in \mathbb{R}$ and $\sigma > 0$ are the location and scale parameters, respectively;
$x \geq \mu$ when $\gamma > 0$ and $\mu \leq x \leq \mu - \sigma/\gamma$ when $\gamma < 0$.

Ref: [5], p.25

Extreme Value Theory – Exceedance & Excess Distributions

Let X be a random variable with a distribution function F and let

$$x_F = \sup\{x \in \mathbb{R}: F(x) < 1\}.$$

For a fixed threshold u,

$$F^{[u]}(x) = \mathrm{P}(X \leq x | X > u) = \frac{F(x) - F(u)}{1 - F(u)}, \qquad x \geq u,$$

is the *exceedance distribution function* of a r.v. X over a threshold u.

The distribution function of a r.v. having GPD is the only continuous distribution function F having *POT-stability* (peaks-over-threshold stability), i.e., such that for a certain choice of constants b_u and a_u

$$F^{[u]}(b_u + a_u x) = F(x).$$

Ref: [5], p.25

Modelling Exceedance Distribution

Distribution function $F^{[u]}(x)$ can be modelled using GPD for large values of u based on the following theorem:

(Balkema-de Haan-Pickands) If $F^{[u]}(x)(b_u + a_u x)$ has a continuous limiting distribution function for $u \to x_F$, then

$$\lim_{u \to x_F} \left| F^{[u]}(x) - W_{\gamma,u,\sigma(u)}(x) \right| = 0,$$

where $\sigma(u) > 0$.

It also holds that

$$\widehat{F_k}(\boldsymbol{y}; \cdot) \approx F^{[u]} \approx W_{\gamma,u,\sigma}, \qquad \text{if } k \text{ and } u \text{ are sufficiently large,}$$

where $\widehat{F_k}(\boldsymbol{y}; \cdot)$ is the sample exceedance distribution function based on the observed exceedances $y_1, \dots, y_k$ over the threshold u.

Therefore, the Generalized Pareto distribution function can be fitted to the sample exceedance distribution function.

 Ref: [5], pp.27-28

Mean Excess Function

Let us define the mean excess function of a r.v. X as follows:

$$e(u) = \mathrm{E}(X - u | X > u).$$

Suppose X has GPD with parameters $\gamma < 1$, $\mu = 0$ and σ. Then for $u < x_F$,

$$e(u) = \frac{\sigma + \gamma u}{1 - \gamma}, \qquad \sigma + u\gamma > 0.$$

Linearity of $e(u)$ can be used to identify an appropriate value of u for the modelling of exceedances.

Ref: [4], p.294

Mean Excess Plot

Let $x_1, \dots, x_n$. be the realizations of r.vs $X_1, \dots, X_n$. Let us denote the ordered observations as $x_{n,n} \leq \cdots \leq x_{1,n}$. For the defined threshold of u, we could construct an empirical estimate $e_n(u)$ of $e(u)$ as follows:

$$e_n(u) = \frac{\sum_{i=1}^{n} (x_i - u) \, \mathrm{I}_{[x_i > u]}}{\sum_{i=1}^{n} \mathrm{I}_{[x_i > u]}}.$$

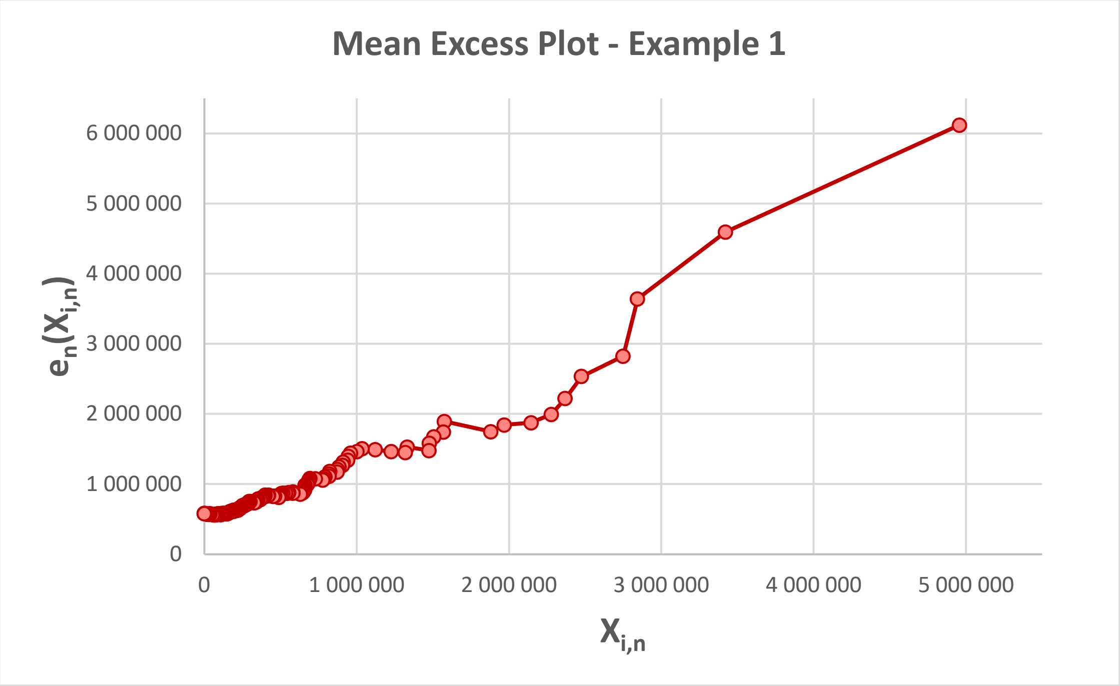
Then, we could plot the following points:

$$\left\{ \left(x_{i,n}, e_n\left(x_{i,n}\right) \right), i = 2, \dots n \right\}.$$

If the observations are following a GPD, this graph should show a linear trend. The point where the shape of the graph becomes linear could serve as an approximation for the threshold of u for modelling exceedances.

Ref: [4], p.296

Mean Excess Plot – Example 1 *(based on randomized claims data)*



There is clearly a linear trend, so any point fulfilling other criteria could be chosen

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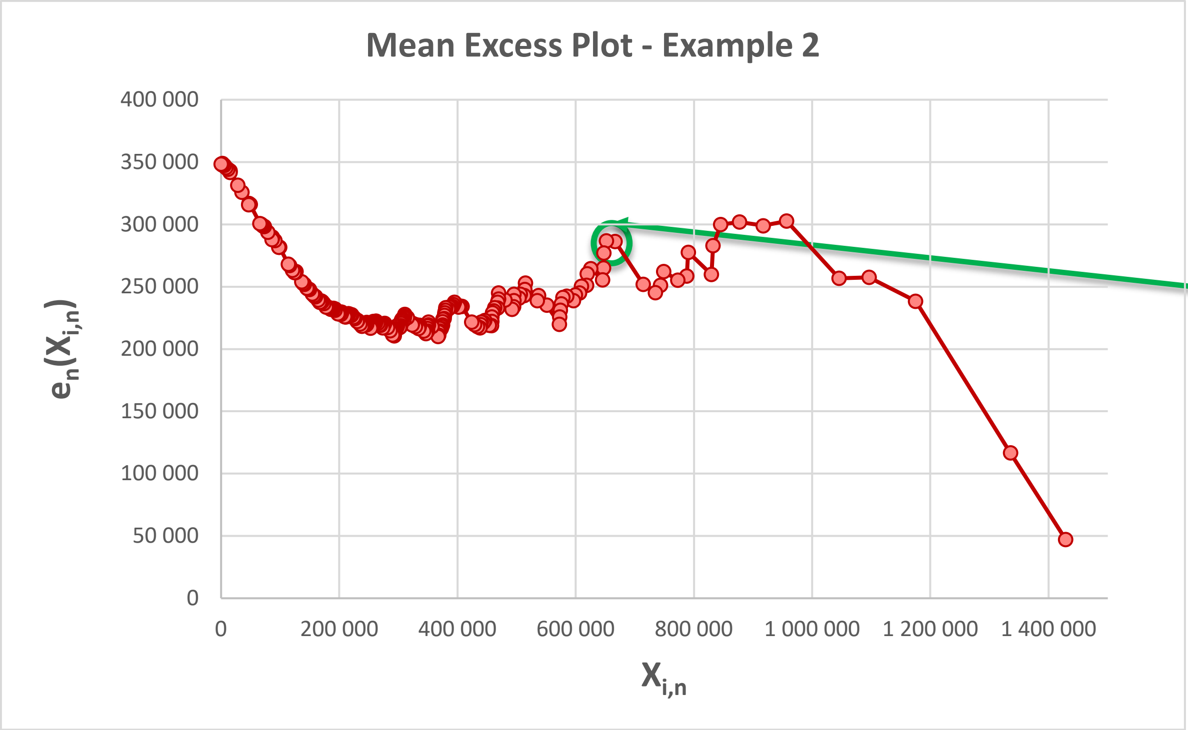
Mean Excess Plot – Example 2 *(based on randomized claims data)*



This point might be a good candidate for the threshold if it fulfils all other criteria

Modelling of MTPL Risks in Reinsurance

Severity Distribution Fitting

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Collective Risk Model

Collective Risk Model constructs aggregate losses from claim size and claim count distributions. We can define aggregate losses in a given year as

$$S = \sum_{i=1}^{N} X_i,$$

where X_i are i.i.d. random variables representing the claim amounts which are independent on the claim count N.

It holds that

$$\mathrm{E}S = \mathrm{E}X \cdot \mathrm{E}N.$$

Using the above relation, we can estimate pure risk premium for an XL reinsurance contract by estimating the parameters of both severity and frequency distributions independently for the observations exceeding a threshold of u equal to the treaty deductible.

Pareto Distribution for Fitting Exceedances

In the previous part of the presentation, we showed that the GPD can be fitted to exceedances over a certain large threshold u. In this presentation we will fit the Pareto distribution which is a special case of the GPD.

The distribution function of a r.v. X following the Pareto distribution with parameter α and threshold of u is defined as follows:

$$F_X(x) = 1 - \left(\frac{x}{u}\right)^{-\alpha} \text{ for } x \geq u \text{ and } \alpha > 0.$$

The above is equivalent to the GP distribution function with location $\mu = u$, scale $\sigma = \frac{u}{\alpha}$ and shape $\gamma = \frac{1}{\alpha}$.

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Maximum Likelihood Estimation of the Pareto Parameter

Let $\boldsymbol{x} = (x_1, \dots, x_n)$ be an observed sample of claim ultimate values (adjusted for inflation development, superimposed inflation, potential IBNER factors etc.). Let u be the value of the threshold defined according to the criteria described earlier, and $\boldsymbol{y} = (y_1, \dots, y_k)$ are the observed exceedances over u.

We will be using Maximum Likelihood method to estimate the parameter α of the Pareto distribution for exceedances $y_1, \dots, y_k$.

The likelihood function for the Pareto distribution parameter α given the exceedances $y_1, \dots, y_k$ and threshold u is defined as follows:

$$\mathcal{L}(\alpha) = \prod_{i=1}^{k} \alpha \frac{u^\alpha}{y_i^{\alpha+1}} = \alpha^k u^{k\alpha} \prod_{i=1}^{k} \frac{1}{y_i^{\alpha+1}}.$$

Maximum Likelihood Estimation of the Pareto Parameter

Therefore, the logarithmic likelihood function is:

$$\ell(\alpha) = k \ln \alpha + k\, \alpha \ln u - (\alpha + 1) \sum_{i=1}^{k} \ln y_i \,.$$

We maximize the log-likelihood function subject to $\alpha > 0$.

To find the estimator for α, we calculate the derivative and determine where it is zero:

$$\frac{\partial \ell}{\partial \alpha} = \frac{k}{\alpha} + k \ln u - \sum_{i=1}^{k} ln\, y_i = 0.$$

Because the second derivative $-\frac{k}{\alpha^2}$ is negative, the following estimator is indeed the MLE for α:

$$\hat{\alpha} = \frac{k}{\sum_i \ln(y_i/u)}.$$

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Pareto Distribution Fitting Example

Let us assume the ultimate claim observations used in Example 1 for the Mean Excess plot. We have chosen 900 000 for the threshold value since

- retention of the XL program the company is buying equals 1 000 000;
- there are 27 observations above the threshold;
- maximum inflated reporting threshold is around 750 000;
- there is a gap between the first claim above the threshold and the last claim below the threshold;
- mean excess plot is linear for the claim values above 900 000.

The rank statistics of the exceedances over the threshold of 900 000 are as follows:

Mean	2 129 207
St. dev.	1 976 185
Min	909 154
25% Quantile	1 226 846
Median	1 506 945
75% Quantile	1 969 477
Max	11 074 683

The estimate of α parameter of the Pareto distribution equals:

$$\hat{\alpha} = 1.54.$$

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Modelling of MTPL Risks in Reinsurance

Frequency & Payout Pattern Estimation

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Poisson Approximation for Exceedances

Let us assume i.i.d. r.vs $X_1, \dots, X_n$ with common distribution function F. Let K be a r.v. determining the number of exceedances of a threshold u between r.vs $X_1, \dots, X_n$. We may write $K = \sum_{i \leq n} \mathrm{I}(X_i > u)$.

Then

$$\mathrm{P}[K = k] = \binom{n}{k} p^k (1-p)^{n-k} =: B_{n,p}(k), \qquad k = 0, \dots, n,$$

where $B_{n,p}$ is the binomial distribution with parameters n and $p = 1 - F(u)$.

The binomial distribution $B_{n,p}$ can be approximated by a Poisson distribution, so K could be regarded as a Poisson r.v. If $np(n) \to \lambda$ as $n \to \infty$, then

$$B_{n,p(n)}(k) \to P_\lambda(k), \qquad n \to \infty,$$

where

$$P_\lambda(k) = \frac{\lambda^k}{k!} e^{-\lambda}, \qquad k = 0,1,2,3, \dots$$

It holds that $\mathrm{E}K = \lambda$

Ref: [5], p.8-9

Frequency Triangulation Development

Let us assume ultimate claim observations $\boldsymbol{x}_{i,n_i} = (x_{i,1}, \dots, x_{i,n_i})$, where $i = 1, \dots m$ are the observed accident years and n_i are numbers of observed claims in accident year i. For each accident year i we denote exceedances of a threshold u by $\boldsymbol{y}_{i,k_i} = (y_{i,1}, \dots, y_{i,k_i})$, where $k_i \leq n_i$ is the number of exceedances in accident year i.

- $\boldsymbol{x}_{i,n_i} = (x_{i,1}, \dots, x_{i,n_i})$ were adjusted for inflation and potential adverse development of known (reported claims).
- however, late reporting of not yet known claims is very common in MTPL;
  - ➢ we need to estimate the triangle development of the exceedances $k_{i,z}$ which stand for the numbers of claims exceeding threshold u in the development year z:

Chain Ladder or Mean Link Ratios could be used

AY/DY	1	...	z
1	$k_{1,1}$	...	$k_{i,z}$
2	$k_{2,1}$	...	
...			

Poisson Distribution Parameter Estimation

Let us assume that $\widetilde{k_1}, \dots, \widetilde{k_m}$ are the numbers of exceedances after the application of the development factors.

Then the ML-estimator of λ for Poisson distribution equals

$$\hat{\lambda} = \frac{1}{m} \sum_{i=1}^{m} \widetilde{k_i} \, ,$$

which is basically the average of exceedances $\widetilde{k_i}$ between all observed accident years $i = 1, \dots m$.

Because the portfolio size of the insurance company may change through time, it is appropriate to assume exceedances relative to exposure units (e.g., number of exceedances per 1 000 vehicles or per 100 000 of premium income).

Poisson Distribution Parameter Estimation

For MTPL specifically it is recommended to use numbers of vehicles (or vehicle-years in case of short-term policies) for the calculation of relative frequencies.

Premium values might not reflect the portfolio changes appropriately because they are influenced by original rate changes.

We assume that $\frac{\widetilde{k_i}}{v_i} \cdot 1\,000$ are relative exceedances per 1 000 vehicles, where v_i represents the number of insured vehicles in an accident year i. Then we can estimate the number of exceedances $\widehat{k_t}$ for the treaty year t as follows:

$$\widehat{k_t} = \widehat{v_t} \cdot \frac{1}{m} \sum_{i=1}^{m} \frac{\widetilde{k_i}}{v_i} \cdot 1\,000,$$

where $\widehat{v_t}$ is the expected number of vehicles for the treaty year t.

The estimate $\widehat{k_t}$ also serves as the estimate of parameter λ of the Poisson distribution.

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Payout Pattern Estimation

In order to estimate the average payout pattern of the claim in various development years, we assume the following:

- For each pair of accident year/development year we calculate the ratio of total paid and total ultimate claims (only for claims exceeding threshold u).
- We will obtain the development triangle of paid percentages for each accident year.
- For each development year we can calculate the average paid percentage based on the accident years' paid percentages.
- It is appropriate to assume weighted average of the paid percentages, where the weights could be e.g. total ultimate claims exceeding threshold u in a given year.
- As the claim history is limited, one could assume certain extrapolation of the historical payout pattern to the value of 100% for the given development period (e.g. 20, 30 or 40 years).
    - ➢ e.g., https://www.actuaries.org.uk/system/files/documents/pdf/crm2-D3.pdf

Payout Pattern Example

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12
2009	0%	11%	15%	36%	41%	45%	49%	55%	67%	70%	89%	95%
2010	1%	7%	15%	27%	44%	47%	52%	59%	77%	81%	87%	
2011	5%	9%	22%	37%	39%	43%	65%	69%	77%	85%		
2012	2%	5%	11%	19%	29%	33%	37%	48%	69%			
2013	4%	12%	17%	33%	45%	56%	66%	71%				
2014	7%	12%	19%	25%	49%	57%	61%					
2015	2%	6%	15%	35%	41%	48%						
2016	0%	0%	19%	31%	33%							
2017	0%	1%	10%	24%								
2018	1%	7%	21%									
2019	4%	18%										
2020	2%											

We assume the average or weighted average of these values

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Reinsurance Contract Pricing Using Simulations

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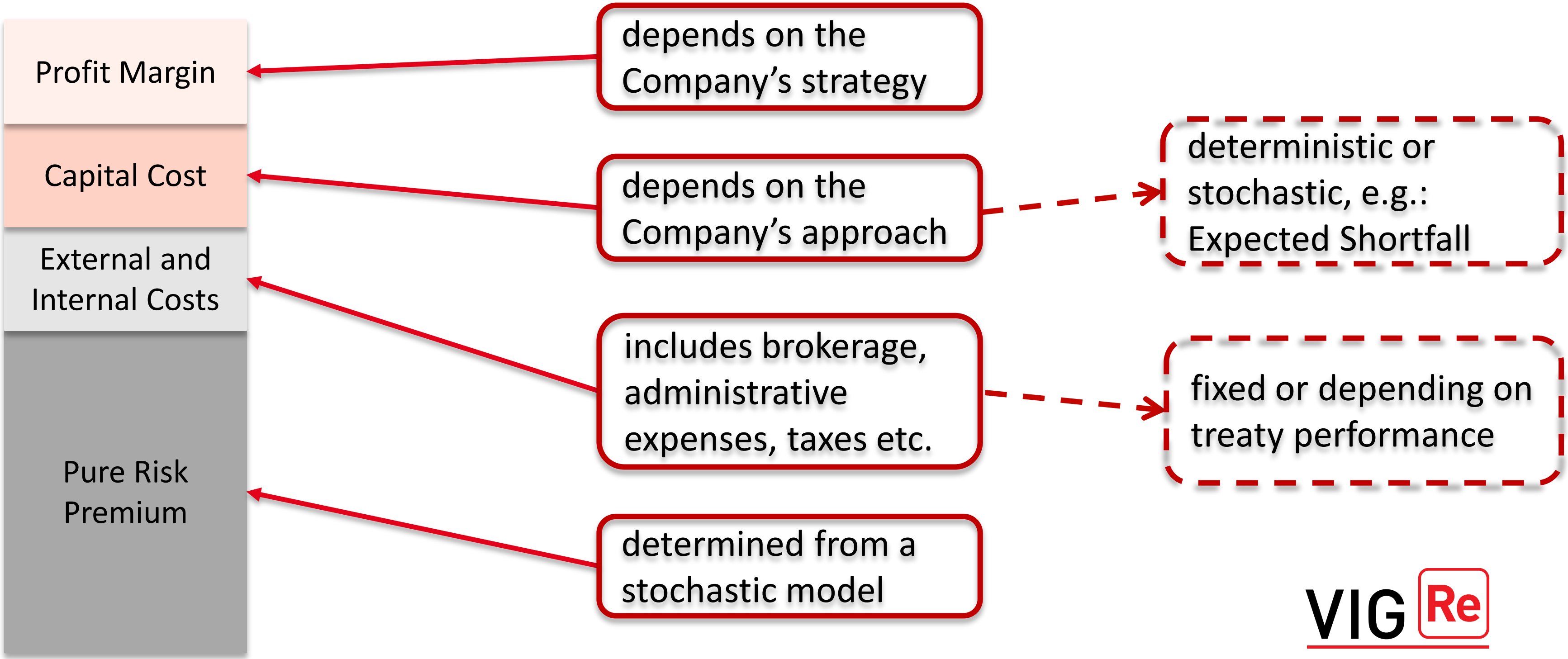
Composition of Reinsurance Price



Frequency Simulation - Homogeneous Poisson Process

Homogeneous Poisson process $N(t)$ with intensity $\lambda > 0$ is a counting process which is defined by the following properties:

a) It starts at zero: $N(0) = 0$ almost surely.

b) It has independent and stationary increments.

c) For every $t > 0$, $N(t)$ is a Poisson r.v. with parameter λt:

$$P(N(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}, \qquad n = 0,1,2, \dots .$$

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Ref: [1], p.485

Poisson Approximation for Exceedances

- It holds that times between events of the homogeneous Poisson process are exponentially distributed with mean $1/\lambda$.
- It can be shown that the point process for exceedances arrivals in a timeframe of $(0,1]$ can be approximated by a homogeneous Poisson process with intensity λ, where mean number of exceedances of threshold u between i.i.d. random variables $X_1, \dots, X_n$ converges to λ.
  - We can approximate arrival times of claims in excess of threshold u by a homogeneous Poisson process with intensity $\hat{\lambda}$ estimated from historical exceedances of threshold u.
  - To simulate the number of threshold exceedances in a given year, we will simulate times between exceedance arrivals in the interval $(0,1]$.

 Ref: [4], pp.238-239

Severity Simulation – Inverse Transform Method

Let us assume a r.v. X with a continuous distribution function F. Let F^{-1} be the quantile function (or the inverse of distribution function F).

We can simply simulate from the distribution function F using the following algorithm:

- Generate a number U from Uniform[0,1] distribution;
- Calculate $x_i = F^{-1}(U)$;
- Repeat until the desired number of simulations n is reached.
    - Can be implemented in various tools, e.g., Excel.

Ref: [3], p.27-28

Inverse Transform Method Example – Pareto Distribution

Let us assume a r.v. X having a Pareto distribution with parameter α and threshold parameter u. Let $\hat{\alpha}$ be the ML-estimator of α. Then, for $x \geq u$ and $\alpha > 0$:

$$F_X(x) = 1 - \left(\frac{x}{u}\right)^{-\alpha}$$

and

$$F_X^{-1}(p) = \frac{u}{(1-p)^{1/\alpha}}.$$

We simulate the values from the Pareto distribution as follows:

- Generate a number p from the Uniform[0,1] distribution;
- Calculate $x_i = \frac{u}{(1-p)^{1/\hat{\alpha}}}$.
- Repeat until the desired number of simulations n is reached.

Steps: Pricing of XL Reinsurance Treaty

1. Data preparation, claims inflation, IBNER factor setting;
2. Threshold setting;
3. Estimation of severity distribution of exceedances;
4. Estimation of average exceedances count for the treaty year;
5. Estimation of payout pattern;
6. Simulating the following treaty year 10 000/100 000+ times;
7. Application of reinsurance XL treaty structure and indexation in each simulation year;
8. Calculation of average ceded loss and its quantiles or other desired cash flows.

XL Treaty Year Simulation

- We assume our exceedances are following Pareto distribution with estimated parameter $\hat{\alpha}$.

- We assume the arrival times of exceedances can be approximated by a homogeneous Poisson process with estimated intensity $\widehat{k_t}$, where $\widehat{k_t}$ is the estimated number of exceedances for treaty year t.

XL Treaty Year Simulation

For each simulation of treaty year t we conclude the following steps:

1. Simulate times between threshold exceedance arrivals from Exponential distribution with parameter $\widehat{k_t}$.

2. Repeat Step 1 until the sum of times between threshold exceedance is larger than 1 (end point of our treaty period).

3. Calculate the count of threshold exceedances (count of times between the exceedances until their sum is larger than treaty period of 1).

4. For each arrival time simulate the respective exceedance severity value (unless there are zero exceedances in the simulation).

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XL Treaty Year Simulation

5. Apply the treaty structure and calculate the ceded loss for each simulated claim.

6. For each simulated claim consider the payout pattern and subsequent indexation/discounting effects.

7. Repeat until the desired number of simulations n is reached.

Example – XL Treaty Result Simulation

*data from Mean Excess Plot – Example 1

- Our client buys unlimited XL reinsurance protection for the losses in excess of 1 000 000 EUR;
- We have chosen a threshold of $u = 900\ 000$;
- Using MLE method, we estimated the parameter of Pareto distribution $\hat{\alpha} = 1.54$;
- Our estimated frequency for the treaty year t equals $\widehat{k_t} = 2.06$.

The rank statistics of claims in excess of 900 000 are as follows:

Mean	2 129 207
St. dev.	1 976 185
Min	909 154
25% Quantile	1 226 846
Median	1 506 945
75% Quantile	1 969 477
Max	11 074 683

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Example – XL Treaty Result Simulation

*data from Mean Excess Plot – Example 1

- **Firstly, we simulate frequency of claims in excess of 900 000 using Poisson process with $\hat{\lambda} = \widehat{k_t} = 2.06$.**

1. Simulation of times between claim arrivals:

Simulation#	T1	T2-T1	T3-T2	T4-T3	T5-T4	T6-T5	T7-T6	T8-T7	T9-T8	T10-T9	T11-T10
1	0,068	0,222	0,858	0,656	0,346	0,286	0,098	0,537	0,238	0,057	0,157
2	0,131	0,137	0,107	0,033	0,667	0,653	2,906	0,710	0,964	0,151	0,142

2. Defining an indicator of whether i-th event has occurred in a given simulation:

Event1	Event2	Event3	Event4	Event5	Event6	Event7	Event8	Event9	Event10	Event11
1	1	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

Example – XL Treaty Result Simulation

*data from Mean Excess Plot – Example 1

3. Calculation of total simulated number of claims in a given simulation year:

Final #events
2
4
3

4. Calculation of mean, standard deviation, empirical quantiles etc.:

Mean	1,97
St. dev.	1,37
Min	0,00
25% Quantile	1,00
Median	2,00
75% Quantile	3,00
Max	8,00

Example – XL Treaty Result Simulation

*data from Mean Excess Plot – Example 1

- Then, for each simulated claim arrival, we simulate the severity value of the corresponding claim from Pareto distribution with $\hat{\alpha} = 1.54$.

Simulation	Loss1	Loss2	Loss3	Loss4	Loss5	Loss6	Loss7	Loss8	Loss9	Loss10	Loss11	Max
1	1 160 236	1 218 201										1 218 201
2	2 440 349	1 782 506	923 588	1 486 223								2 440 349

Rank statistics of 1st simulated claim

Mean	2 450 086
St. dev.	5 524 443
Min	900 881
25% Quanti	994 238
Median	1 314 641
75% Quanti	2 097 642
Max	145 132 284

Rank statistics of Max claim in a given simulation

Mean	3 269 606
St. dev.	6 766 144
Min	0
25% Quanti	1 161 642
Median	1 886 350
75% Quanti	3 231 237
Max	145 132 284

Example – XL Treaty Result Simulation

*data from Mean Excess Plot – Example 1

- **For each simulated claim severity, we calculate the ceded reinsurance loss as follows:**

Ceded_Loss1	*Ceded_Loss2*	*Ceded_Loss3*	*Ceded_Loss4*	*Ceded_Loss5*	*Ceded_Loss6*	*Ceded_Loss7*	*Ceded_Loss8*	*Ceded_Loss9*	*Ceded_Loss10*	*Ceded_Loss11*	**Sum**
160 236	218 201	0	0	0	0	0	0	0	0	0	378 437
1 440 349	782 506	0	486 223	0	0	0	0	0	0	0	2 709 078
21 504 718	116 850	2 486	0	0	0	0	0	0	0	0	21 624 054

Rank statistics of total reinsurance loss

Mean	2 913 334
St. dev.	7 161 440
Min	0,00
25% Quantile	194 048
Median	1 151 153
75% Quantile	3 024 515
Max	145 301 754

Pure reinsurance premium is **2 913 334**

Should be loaded by other components of the reinsurance price

Simulation from the Model vs History

*data from Mean Excess Plot – Example 1

- **To ensure that our model performs in line with claim history, we calculate pure reinsurance premium based on historical Burning Costs:**

AY	Ceded Loss	Exposure	Burning Cost
2002	1 602 397	715 913	1,32%
2003	0	742 301	0,00%
2004	1 476 026	785 071	1,11%
2005	0	809 953	0,00%
2006	569 810	846 788	0,40%
2007	3 956 482	856 105	2,72%
2008	0	871 416	0,00%
2009	3 829 213	881 109	2,56%
2010	0	904 522	0,00%
2011	0	945 476	0,00%
2012	475 865	990 289	0,28%
2013	226 846	1 055 621	0,13%
2014	10 111 665	1 094 113	5,44%
2015	1 360 679	1 132 930	0,71%
2016	5 264 300	1 190 432	2,60%
2017	1 945 229	1 232 988	0,93%
2018	0	1 301 885	0,00%
2019	0	1 354 501	0,00%
2020	0	1 408 387	0,00%

Calculated as:

$$\frac{Ceded\_Loss_{2002}}{Exposure_{2022} \cdot Average\_Premium_{2022}}$$

Avg. Premium 2022	170
Premium 2022	259 471 217

Average Claim 2002-2018	1 812 854
Average BC 2002-2018	1,07%

Calculated as:

$$Average\ BC \cdot Premium_{2022}$$

Pure reinsurance premium is **2 773 993**

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Modelling of MTPL Risks in Reinsurance

Index Clause Impact on Reinsurance

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Why Indexation?

It takes many years to settle bodily injury claims.

- Future claim payments would increase due to wage inflation, increased medical/nursing costs etc.
- XL treaty deductible should be adjusted accordingly to reflect the increase in payments.

The main purpose of the Index Clause is

- to maintain a stable relationship between the reinsurance retention and the reinsurance limit;
- to ensure that the relative proportion of loss distributed between the reinsured limit and retention would stay the same.

Indexation – Important Considerations

Index Clause defines the method for reinsurance deductible and limit adjustment in case of major fluctuations in a certain base index.

Bodily injury claims are the main contributors of XL treaty claims.

- It is appropriate to assume wage index or a blended index including wage and medical cost index, since these are the main factors responsible for the increase in claim value.
- The type of index to be used for indexation is stated in the Index Clause in the reinsurance slip.
- In practice, wage index is often used for the indexation according to slip wording, however, in some cases CPI index should be considered (but it is not properly reflecting the potential changes in claim value).

Indexation – Important Considerations

- **Base year** – year/certain date from which we start tracking the index changes.
  - Usually, the average index for 4 quarters preceding the inception of a reinsurance contract should be used.
  - Sometimes the base year is defined in a different way (e.g., wage index from 2 years back or more) – it is important to read the conditions of the Index Clause!!
- In countries with high(er) superimposed inflation the Index Clause is usually not compensating the Reinsurer for price fluctuation in full.
- In case the XL cover is not unlimited (e.g., for Domestic claims in certain countries), reinsurance limit is indexed accordingly as well.
- **Only bodily injury claims are indexed in most treaties!**

Types of Index (Stability) Clauses

- **Full Index Clause**
  - Applied to the full amount of the cost increase, from the very first year of increase in the index.
- **Severe Inflation Clause**
  - Applied only to the amount in excess of a certain level of inflation.
  - Higher levels (generally 30%) are considered for the inflation threshold.
- **Franchise Index Clause**
  - The most frequent Index Clause in European MTPL.
  - Starts working only if the rate of inflation exceeds base index by a certain value (the franchise) but is then applied to the entire claims inflation.
  - Mostly, 10% Franchise is considered.

Severe Inflation Clause Example

Index Clause (30% Excess)

(A) In the event of any loss hereunder the Retention and Limit of the Reinsured shall be adjusted by reference to an Index, as hereinafter defined, for the period embracing the 1st March 2020, in the manner hereinafter set out. The Index for the period embracing the above mentioned date shall be called the BASE INDEX. There shall be no adjustments to the Retention of the Reinsured until the increase in the BASE INDEX exceeds 30% (thirty per cent).

(B) In respect of any loss settlement(s) made under this Contract, the Reinsured shall submit a list of payments comprising such loss settlement(s) showing the Amount(s) of Payment and the Date(s) of Payment. All payments made by the Reinsured in respect of a Bodily Injury claim relating to a Claimant, including the Claimant's legal costs and legal costs incurred by the Reinsured in the defence of a claim, shall be aggregated and the Index used shall be that for the period embracing the Date of Payment, as defined below. However, continuing regular payments shall be treated as if they were separate payments.
The amount of each such payment shall be adjusted by means of the following formula:

$$\frac{\text{Amount of Payment} \times \text{BASE INDEX} \times 1.30}{\text{Index for the period embracing the Date of Payment}} = \text{Adjusted Payment Value}$$

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Franchise Margin Clause Example

2) In the event of any loss hereunder the Underlying Loss of the Reinsured and the Limit of Indemnity of the Reinsurers shall be adjusted by reference to an index, as hereinafter defined, applying at the month of January in the manner hereinafter set out. The index of the above-mentioned date shall be called the Base Index, being the average index for the country/countries of any claimant for the 4 quarters preceding the inception of the Contract.

3) In respect of each and every loss settlement made under this Contract, the Reinsured shall submit a list showing in respect of each and every loss, any settlement payments, indicating the amount(s) paid and the date(s) of payment, as well as any separately calculated loss reserve. The sum of each and every loss settlement payment, if any, plus the current loss reserve, if any, shall for the purpose of this clause be termed the "Actual Amount of Loss Development".

4) The "Adjusted Amount of Loss Development" shall be calculated individually for each and every loss settlement by the following means:

(i)	Paid Loss on Day "X"	x	Base Index (Index on Day "X")	=	Adjusted Paid Loss
(ii)	Current O/S Loss	x	Base Index (Index on Current Day)	=	Current Adjusted O/S Loss
(iii)	All Adjusted Paid Losses + Current Adjusted O/S Loss			=	Adjusted Amount of Loss Development

(where Index on Day "X" and Index on Current Day means average quarterly index valid on the date of the respective payment and/or corresponding reserve, if any). Average quarterly index is calculated as an average of quarterly index valid on the date of the respective payment and/or corresponding reserve, if any, and indexes of three preceding quarters.

However, the above calculation shall only apply in respect of those loss developments where there is a variation of more than 10% (ten percent) compared with the base index. In respect of all other loss developments the "Adjusted Amount of Loss Development" shall always be equal to the "Actual Amount of Loss Development".

The Underlying Loss of the Reinsured and the Limit of Indemnity of the Reinsurers shall then be multiplied by the following fraction:

$$\frac{\text{Actual Amount of Loss Development}}{\text{Adjusted Amount of Loss Development}}$$

In consequence of the above indexation calculation, the Underlying Loss and the Limit of Indemnity re-established could exceed the amounts as set out above.

Common Index Clauses in Europe

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COMPARISON OF INDEXATION CLAUSES PER TERRITORY

Country	LMIC/European	Margin Type	Margin	BI/PD	Type
Austria	European	Franchise or SIC	10%/20%	BI & PD	1 or 2
Belgium	European	Franchise	10%	BI & PD	1
France	European	Franchise	10%	BI & PD	1
Denmark	European	Franchise	10%	BI & PD	1
Germany	European	Franchise or SIC	10%/20%/30%	BI & PD	1 or 2
Italy	European or LMIC	Franchise	10%	BI & PD (European)/ BI Only (LMIC)	1 or 4
Luxembourg	European	Franchise	10%	BI & PD	1
Netherlands	European	SIC	10%	BI only	3
Poland	European	Franchise	10%	BI & PD	1
Portugal	European	Franchise	15%	BI & PD	1
Spain	European	Franchise	10%	BI & PD	1
Sweden	European	SIC	10%/20%	BI & PD	2
Switzerland	European	Franchise	10%	BI & PD	1
UK	LMIC	Full Index	Nil	BI only	5

Type Codes

1 = European Franchise PD/BI 2 = European SIC PD/BI 3 = European SIC BI only 4 = LMIC Franchise BI only 5 = LMIC Full Index

EIC
Net present value, based on the payment-weighted average of each payment and index value at each date of payment

London Market Indexation Clause (LMIC)
Net present value calculated on basis of index at date of the most recent payment, and finally, settlement (With exception of payments subject to UK Courts Act 2003)

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*source: https://www.guycarp.com/insights/2008/09/indexation-clauses-in-liability-reinsurance-treaties-a-comparison-across-europe.html

Impact of Index Clauses on Reinsurance Loss

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Effects of Different Types of Indexation Clause

Layer Attachment	10,000,000
Layer Limit	10,000,000
Index annual growth rate	4%

	Gross Loss									
Year	Cumulative	Incremental	No Index	Full Index (European)	SIC 10% (European)	SIC 20% (European)	Full Index (LMIC)	SIC 10% (LMIC)	SIC 20% (LMIC)
1	2,000,000	2,000,000	-	-	-	-	-	-	-
2	4,000,000	2,000,000	-	-	-	-	-	-	-
3	6,000,000	2,000,000	-	-	-	-	-	-	-
4	8,000,000	2,000,000	-	-	-	-	-	-	-
5	10,000,000	2,000,000	-	-	-	-	-	-	-
6	12,000,000	2,000,000	2,000,000	776,554	1,591,926	1,944,964	-	720,478	1,660,439
7	14,000,000	2,000,000	4,000,000	2,563,808	3,421,538	3,850,836	1,096,227	2,269,298	3,246,856
8	16,000,000	2,000,000	6,000,000	4,348,518	5,242,824	5,732,154	2,580,076	3,800,069	4,816,730
9	18,000,000	2,000,000	8,000,000	6,130,689	7,057,813	7,596,715	4,043,279	5,312,072	6,369,400
10	20,000,000	2,000,000	10,000,000	7,910,328	8,867,718	9,449,078	5,485,011	6,804,555	7,904,175

Notes
1) Claim occurs six months after contract inception
2) Claim assumed to be fully valued at DOL (with no subsequent incurred cost development)
3) Ten equal payments; first payment at DOL, then nine subsequent payments at yearly intervals
4) Index assumed to start at contract inception, with a 4% annual growth rate. Index assumed to be reported at end of each month

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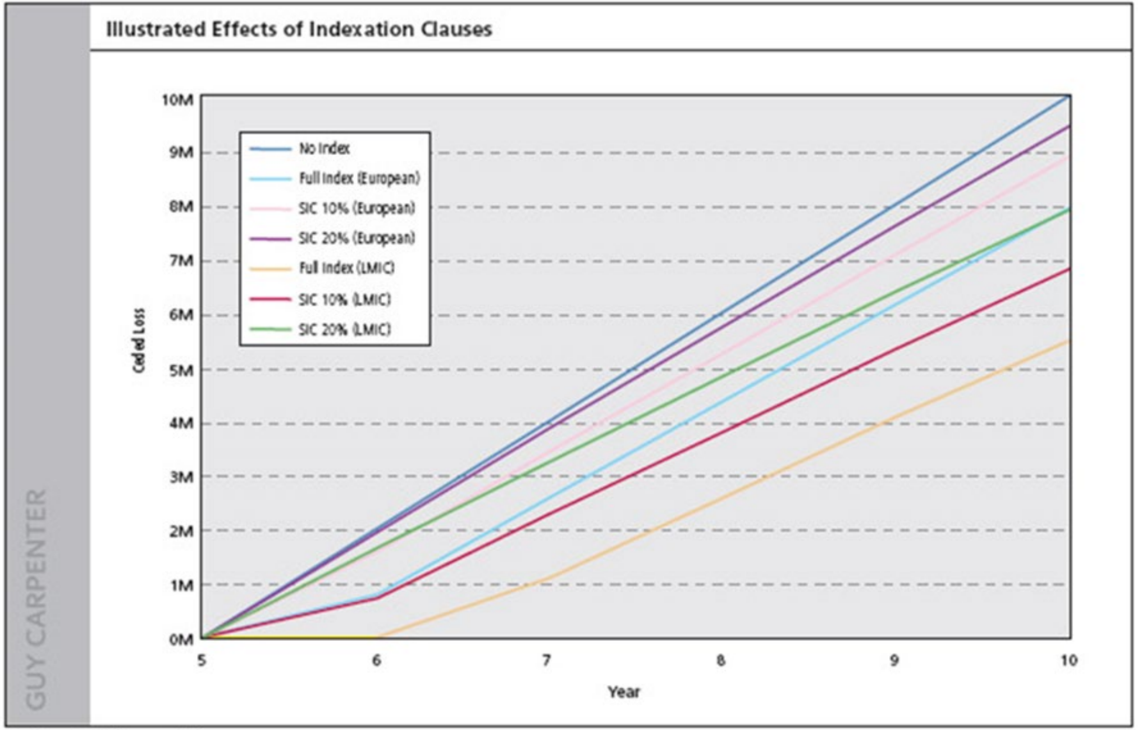
*source: https://www.guycarp.com/insights/2008/09/indexation-clauses-in-liability-reinsurance-treaties-a-comparison-across-europe.html

Impact of Index Clauses on Reinsurance Loss



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*source: https://www.guycarp.com/insights/2008/09/indexation-clauses-in-liability-reinsurance-treaties-a-comparison-across-europe.html

Index Clause in Pricing

- **Index Clauses have a significant impact on MTPL pricing.**
  - ➢ Indexation effect should be considered according to the Indexation Clause described in the Reinsurance Treaty Wording.
- **The effect is more visible in case of higher inflation and/or long claim settlement (payout pattern).**
- **Estimated payout pattern should be used to calculate the impact of Index Clause on future claim payments.**
- **Projections of wage index should be used.**
  - ➢ As claims are being settled within 20-30 years or more, it is crucial to estimate future wage inflation properly.

Index Clause: Stochastic Approach

- As we simulate the possible outcomes of a treaty year, we consider the ultimate claim value(s) in a given simulation.
- We also consider our estimates of the payout pattern and future inflation.
- By applying the payment schedule to each simulated claim, we can calculate the indexed deductible and limit for each year of the claim settlement.
- We calculate the final ceded loss by applying the indexed deductible and limit from the year when the claim was fully settled.
- Discounting of future payments to the present value could be also considered.

Conclusion

- **MTPL pricing in Reinsurance is a complex task, where many factors should be taken into account.**
- **Pure reinsurance premium is hugely influenced by**
  - Assumptions on historical and future inflation;
  - Assumptions on exposure of Insurance Company for the next year;
  - Superimposed inflation assumption;
  - Indexation effect;
  - Potential changes in portfolio (different portfolio composition or client profile, competition in respect of original premiums);
  - Known changes in legislation;
  - Known changes of reserving methods considered by Insurance Company.
- **It is important that the Pricing Actuary is aware about market specifics which should be considered in treaty pricing.**

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Thank you for your attention!

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