Riziko rezerv na jednoletém horizontu

Seminář aktuárských věd, 8. listopadu 2013

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TOOLS 4F

WE UNDERSTAND YOUR JOB

Náplň přednášky

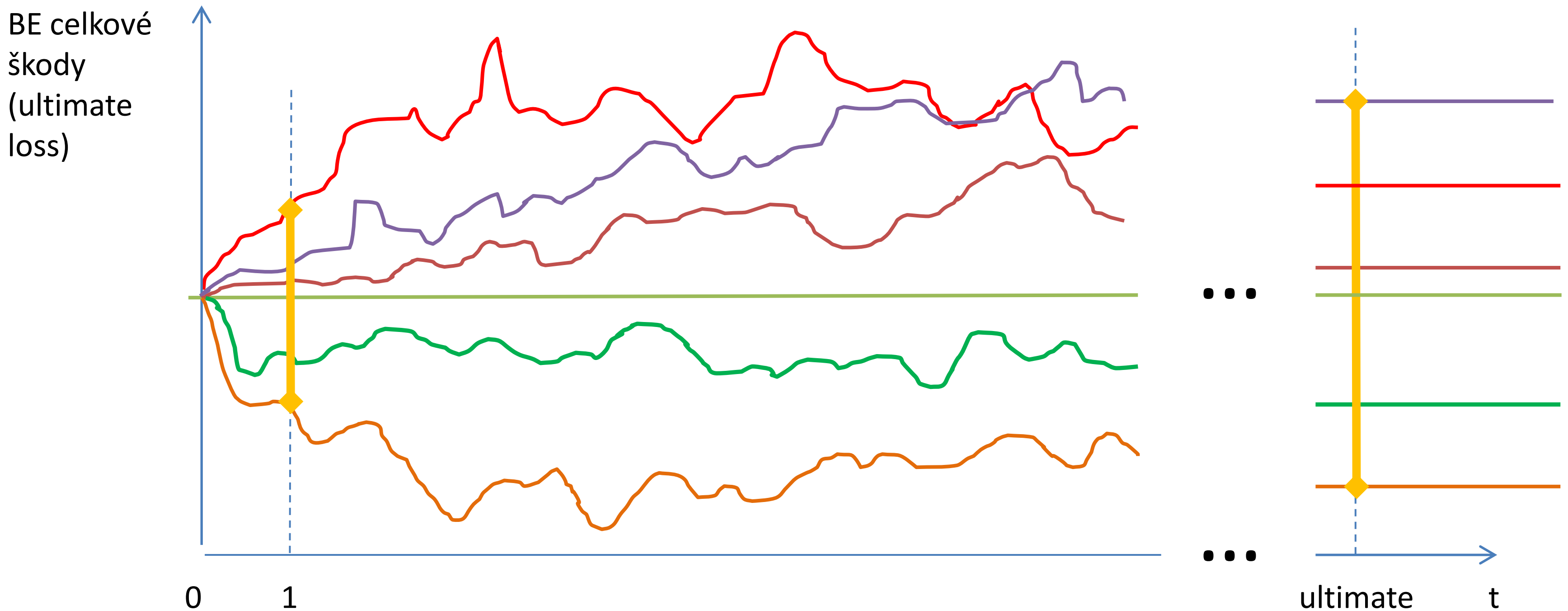
- Jednoletý vs. „ultimate“ horizont
- BE budoucích závazků jako stochastický proces
- Přístupy k modelování jednoletého rizika
- Bootstrap – ukázkový příklad

Jednoletý vs. „ultimate“ horizont

„Ultimate" horizont

- Tradiční přístup
- Výpočet technických rezerv jako best estimate budoucích závazků a odhad variability (míry rizika) této rezervy
- Kvantifikace rizika toho, že vytvořená technická rezerva nepokryje budoucí výplaty pojistných plnění
- Stanovení rezervy podle zvolené hladiny spolehlivosti

- Analyzovanou náhodnou veličinou je rozdíl $BE^{\text{ultimate loss}}(t) - BE^{\text{ultimate loss}}(0)$
- Nejčastěji používanou mírou rizika je VaR (99.5% kvantil)
- Standardní případ: VaR(1-year) < VaR(ultimate)



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k-letý horizont

- Strategické a taktické plánování
  - srovnání dostupného kapitálu a budoucích závazků pro různé horizonty a různé hladiny spolehlivosti
- Udržení solventnosti na víceletém horizontu
- Strategie zajištění
- Alokace aktiv
- Tvorba produktů – např. nastavení výše spoluúčasti

- Jednoletý horizont: SCR podle Solvency II

Proces vypořádání pojistných událostí

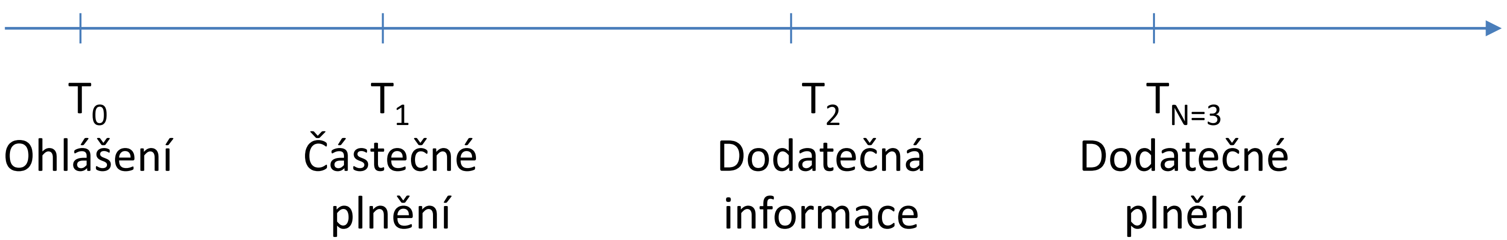
„Handling times“

$(T_j)_{j\geq 0}$

T_0 - ohlášení škody;

$T_1 < T_2 < \ldots < T_N$ - výplaty/nové informace;

$T_{N+1} = T_{N+2} = \ldots = \infty.$

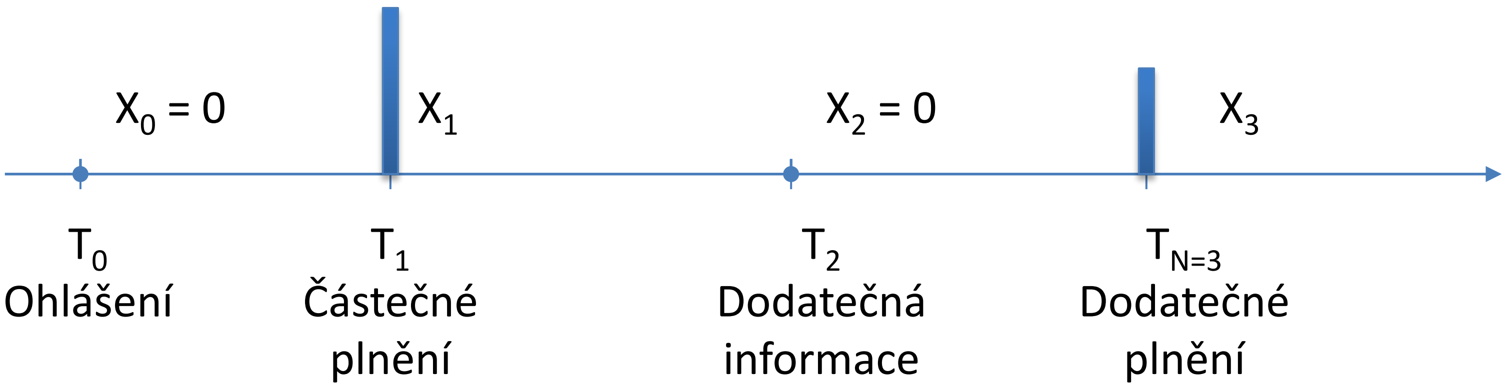


„Payment process"

$$(T_j, X_j)_{j \geq 0}$$

$$\mathrm{X_j} \geq 0 \text{ - výplata v čase } \mathrm{T_j};$$

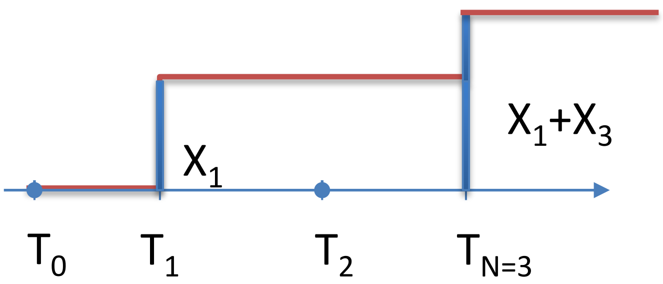
$$X_{N+1} = X_{N+2} = \ldots = 0.$$



„Payment process“

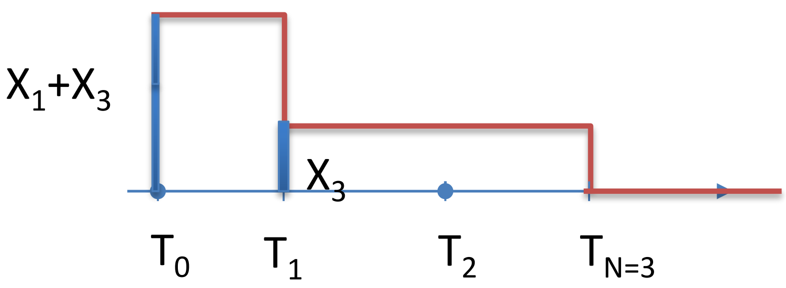
$$(X(t)):\ X(t) = \sum_{j:\,T_j \le t} X_j$$ ... kumulativní výplaty

- rostoucí skoková funkce
- $X(t) = 0$ pro $t < T_0$
- $X(\infty) = \lim (X(t), t \to \infty) = \sum_{j \ge 0} X_j$



$$(U(t)):\ U(t) = X(\infty) - X(t) = \sum_{j:\,T_j > t} X_j$$ ... budoucí závazky

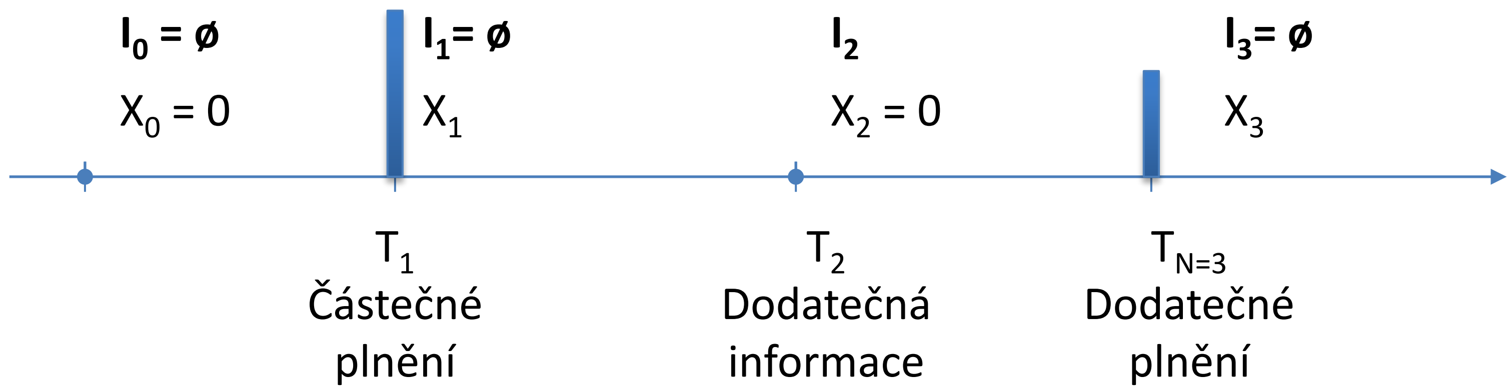
- klesající skoková funkce
- $U(t) = X(\infty)$ pro $t < T_0$
- $\lim (U(t), t \to \infty) = 0$



„Settlement process“

$$(T_j,(X_j,I_j))_{j\geq 0}$$

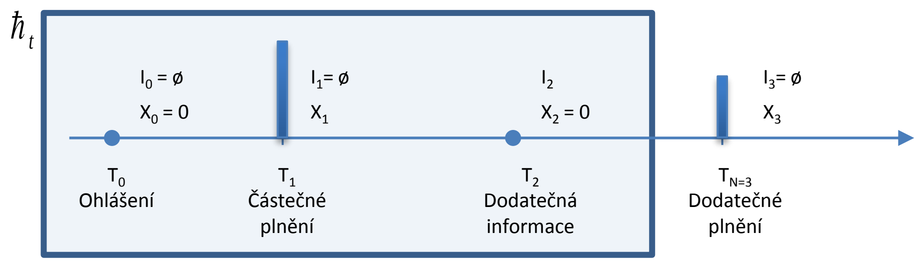
I_j - nová informace v čase T_j;



„Prediction process“

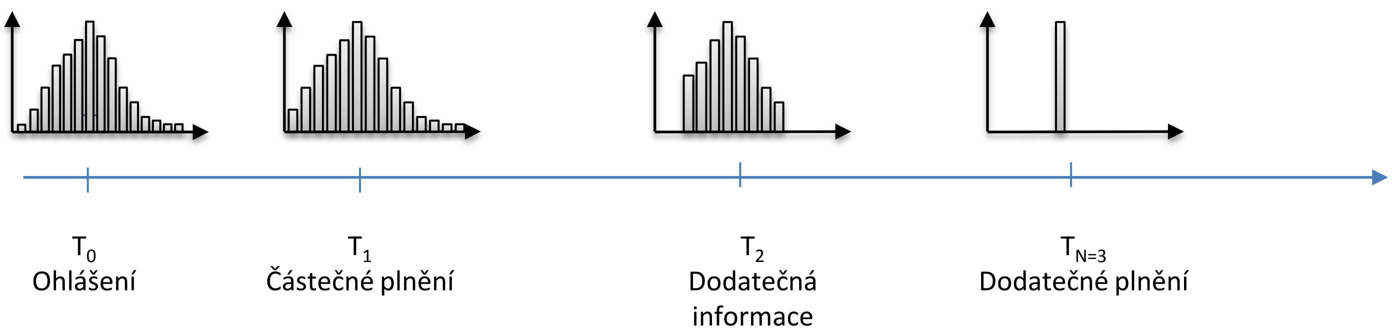
$$(\mu_t): \mu_t(\bullet) = P(X(\infty) \in \bullet \mid \hbar_t)$$

$$\hbar_t = \{(T_j, X_j, I_j)_{j\geq 0}, T_j < t\} \ldots \text{informace dostupné v čase t}$$



$$(M_t): M_t = E^{\hbar_t}(X(\infty))$$

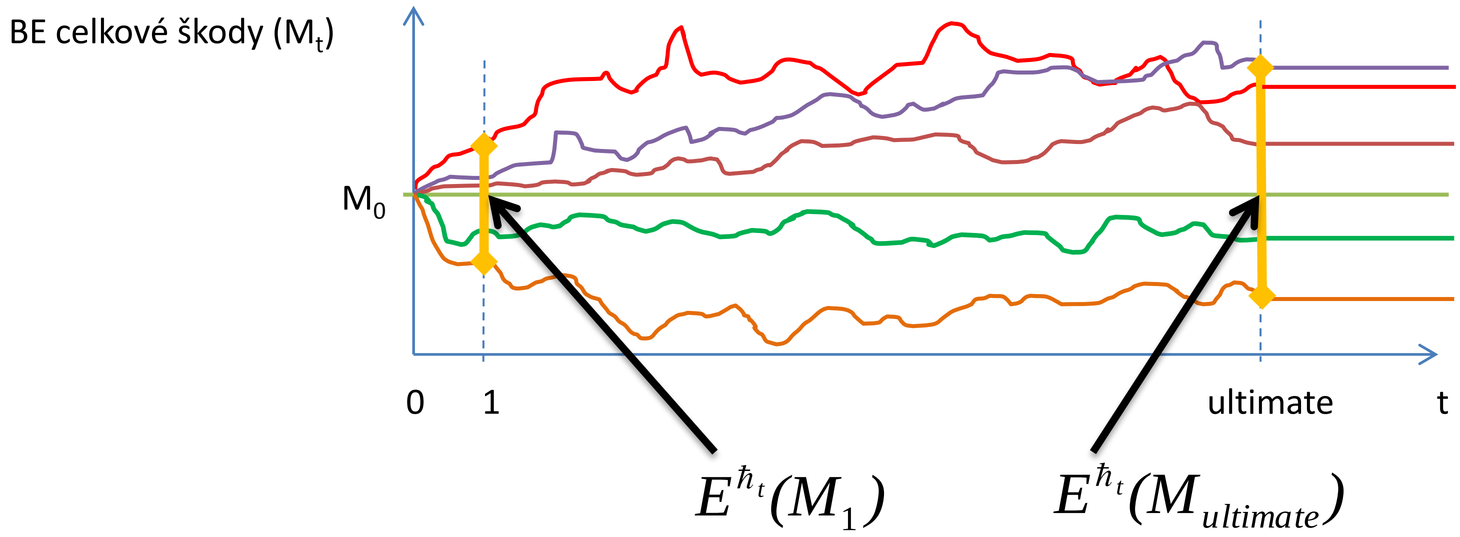
$$(V_t): V_t = Var^{\hbar_t}(X(\infty))$$



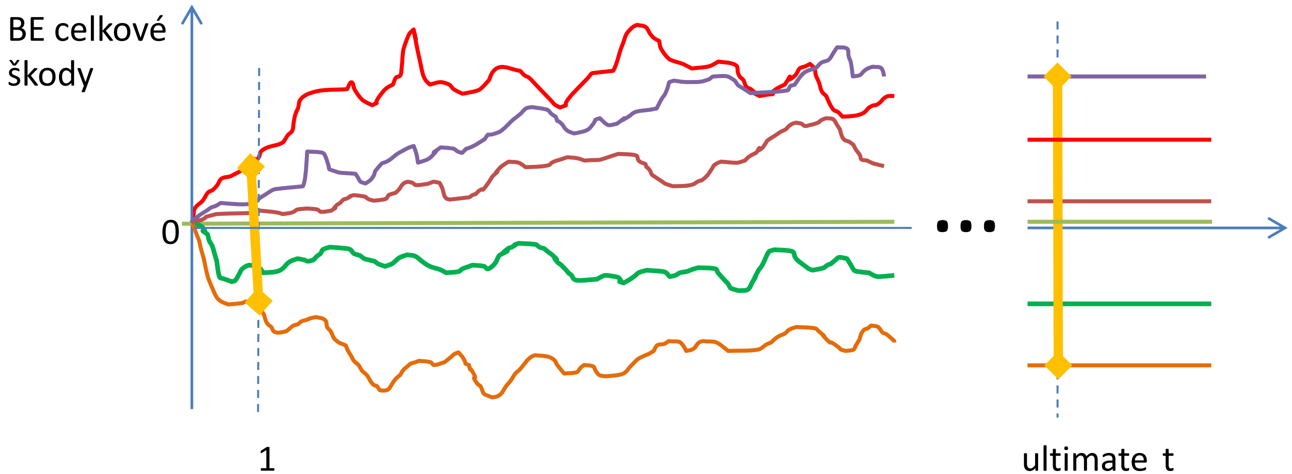
Podmíněná střední hodnota (BE) celkové škody (ultimate loss)

martingalová vlastnost: $E^{\hbar_t}(M_u) = M_t,\ t < u$

$\Rightarrow$ **aktuální odhad budoucího odhadu celkové škody je roven aktuálnímu odhadu celkové škody**



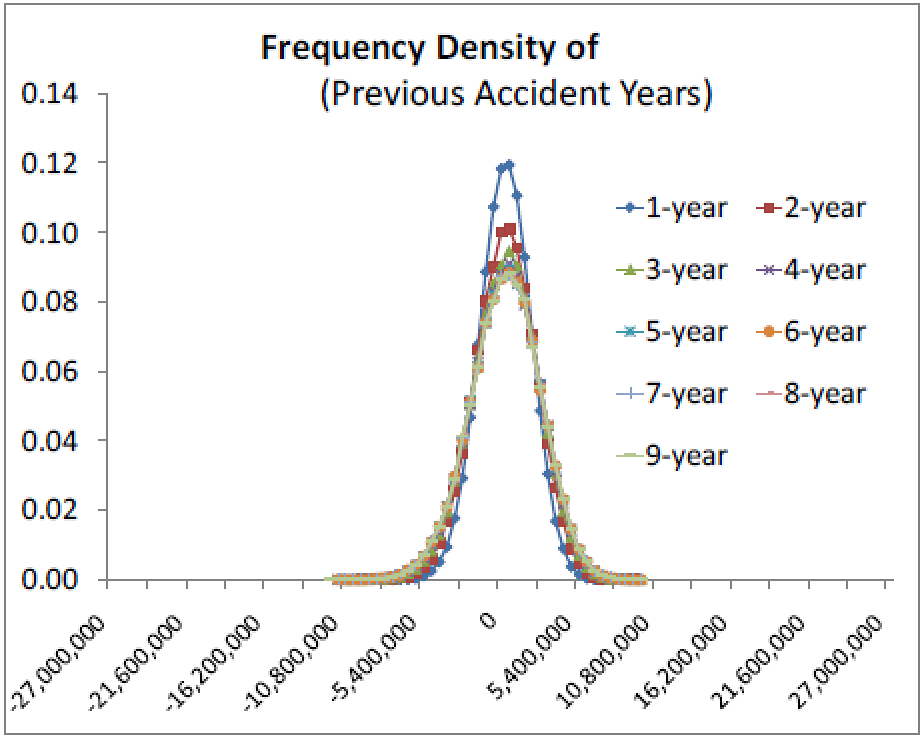
Změna BE celkové škody

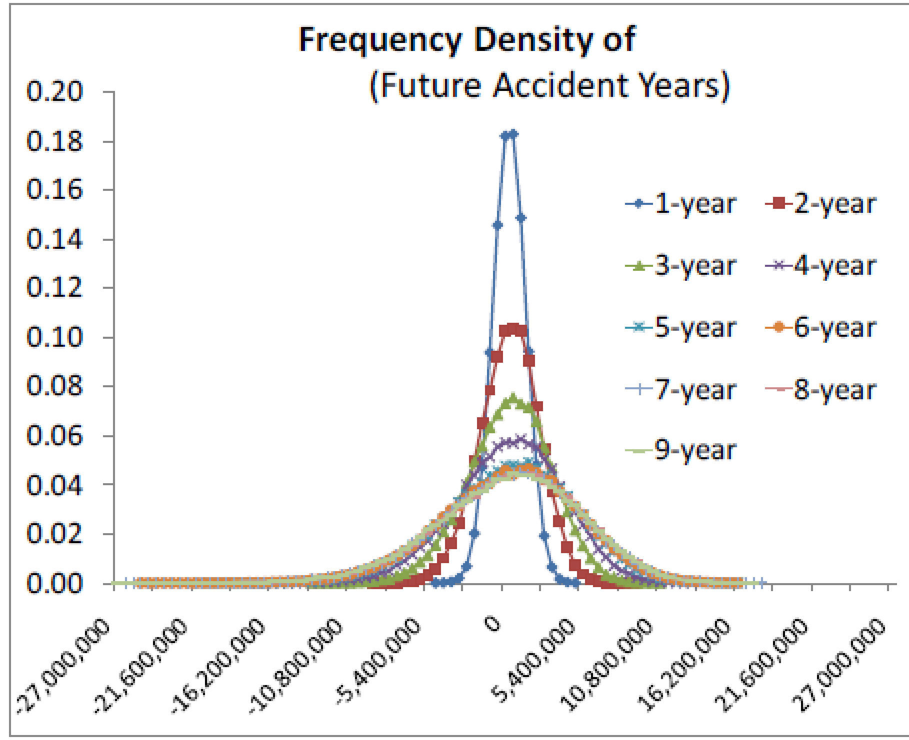


$$E[M(t,t+k)] = 0$$

$$Var[M(t,t+k)] = ?$$

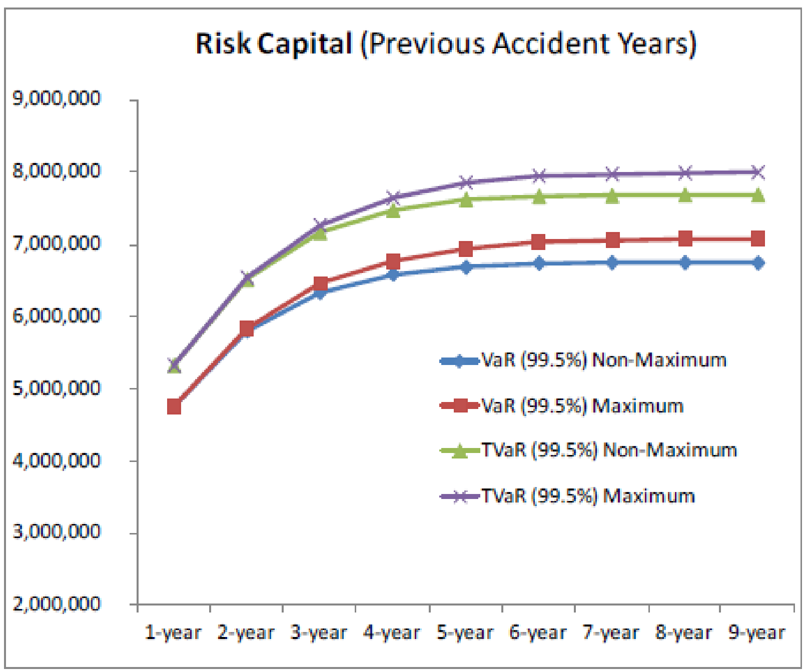
$$VaR[M(t,t+k)] = ?$$





Zdroj: Dorothea Diers, Martin Eling, Christian Kraus, Marc Linde, (2013) "Multi-year non-life insurance risk", Journal of Risk Finance, The, Vol. 14 Iss: 4, pp.353 - 377

Year	Previous Accident Years			
	$VaR^{max}_{99.5\%}$	$VaR^{max}_{99.8\%}$	$TVaR^{max}_{99.5\%}$	$TVaR^{max}_{99.8\%}$
1-year	4,749,386	5,316,952	5,286,335	5,823,192
2-year	5,829,230	6,535,172	6,487,012	7,168,015
3-year	6,453,611	7,258,944	7,226,344	7,972,264
4-year	6,762,882	7,636,010	7,628,222	8,397,742
5-year	6,927,992	7,850,977	7,889,427	8,642,606
6-year	7,027,061	7,941,547	7,950,906	8,737,128
7-year	7,057,542	7,969,071	7,980,294	8,763,660
8-year	7,069,649	7,986,028	7,987,464	8,784,452
9-year	7,072,591	7,993,424	7,992,291	8,795,100



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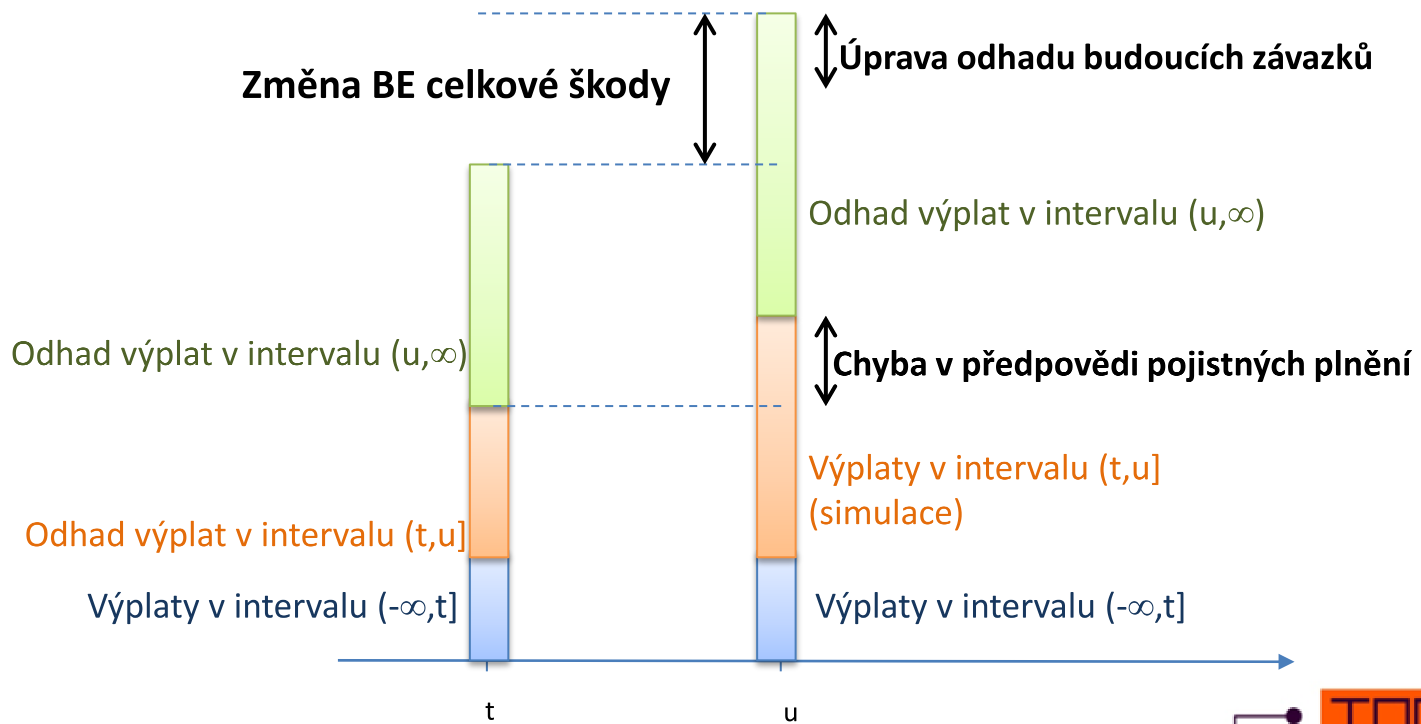
Změna BE celkové škody

$$t < u : M(t,u) = M_u - M_t, X(t,u) = X(u) - X(t)$$

$$M(t,u) = [X(t,u) - E^{\hbar_t}(X(t,u))] + [E^{\hbar_u}(U_u) - E^{\hbar_t}(U_u)]$$

chyba v předpovědi pojistných plnění v intervalu (t,u]

úprava odhadu budoucích závazků (tj. v intervalu (u,∞)) na základě informací získaných v intervalu (t,u]



Metody odhadu jednoletého rizika

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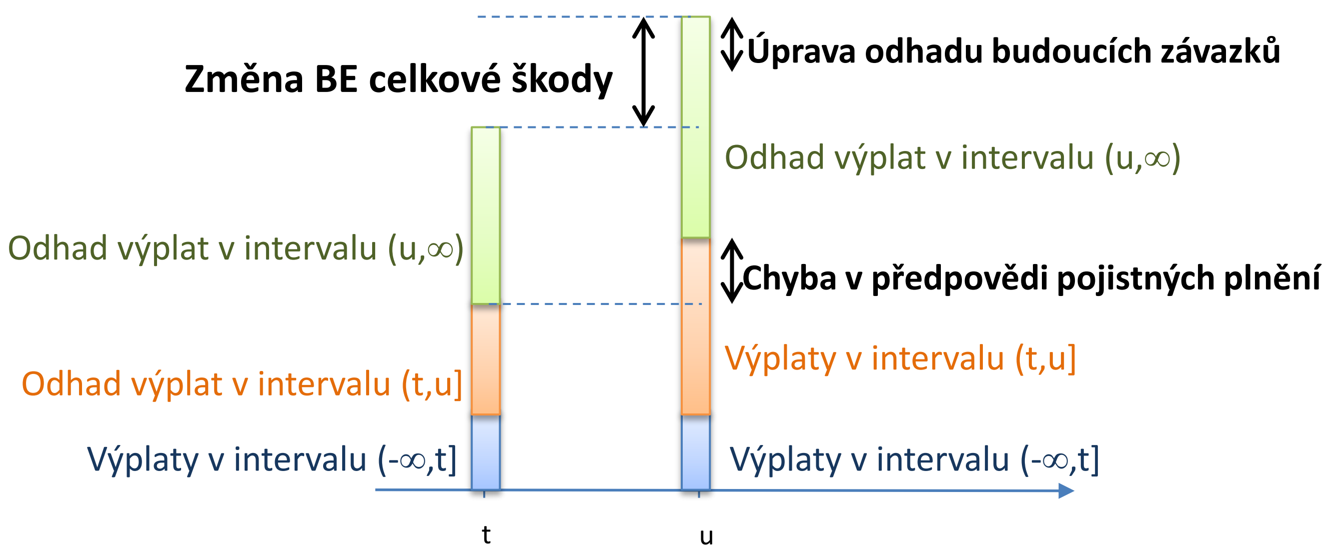
Claims development result

V literatuře se změna BE celkové škody obvykle označuje jako „claims development result“ (CDR)

$$CDR(t,t+1) = BE(t) - [BE(t+1) + Claims(t,t+1)]$$



Analytická formule

- Wuthrich, Merz, Lysenko
- Založeno na Mackově modelu (Chain ladder) => nutnost splnění předpokladů modelu
- Rozklad rizika: $\mathrm{Var}\left(\sum_{i=1}^{I} \mathrm{CDR}_i(I+1) \,\middle|\, \mathcal{D}_I\right)$ + $\mathrm{MSE}_{\mathcal{D}_I}\left(\sum_{i=1}^{I} \widehat{\mathrm{CDR}}_i(I+1)\right)$

  Process error — Estimation error

$$\widehat{\mathrm{Var}}\left(\sum_{i=1}^{I}\widehat{\mathrm{CDR}}_i(I+1)\middle|\mathcal{D}_I\right)=\sum_{i=1}^{I}\widehat{\Gamma}_{i,J}^{I}+2\cdot\sum_{i>k>0}\widehat{\Upsilon}_{i,k}^{I}, \tag{3.16}$$

where for $i \geq 1$

$$\widehat{\Gamma}_{i,J}^{I}=\widehat{\mathrm{Var}}\left(\widehat{\mathrm{CDR}}_i(I+1)\middle|\mathcal{D}_I\right)$$

$$=\left(\widehat{C}_{i,J}^{I}\right)^2\cdot\left\{\left(\left[1+\frac{\left(\widehat{\sigma}_{I-i}^{I}\right)^2/\left(\widehat{f}_{I-i}^{I}\right)^2}{C_{i,I-i}}\right]\cdot\prod_{l=I-i+1}^{J-1}\left(1+\frac{\left(\widehat{\sigma}_{l}^{I}\right)^2/\left(\widehat{f}_{l}^{I}\right)^2}{\left(S_l^{I+1}\right)^2}\cdot C_{I-l,l}\right)\right)-1\right\}, \tag{3.17}$$

and for $i > k > 0$

$$\widehat{\Upsilon}_{i,k}^{I}=\widehat{\mathrm{Cov}}\left(\widehat{\mathrm{CDR}}_i(I+1),\widehat{\mathrm{CDR}}_k(I+1)\middle|\mathcal{D}_I\right)$$

$$=\widehat{C}_{i,J}^{I}\cdot\widehat{C}_{k,J}^{I}\cdot\left\{\left(\left[1+\frac{\left(\widehat{\sigma}_{I-k}^{I}\right)^2/\left(\widehat{f}_{I-k}^{I}\right)^2}{S_{I-k}^{I+1}}\right]\prod_{l=I-k+1}^{J-1}\left(1+\frac{\left(\widehat{\sigma}_{l}^{I}\right)^2/\left(\widehat{f}_{l}^{I}\right)^2}{\left(S_l^{I+1}\right)^2}\cdot C_{I-l,l}\right)\right)-1\right\}. \tag{3.18}$$

$$\widehat{\mathrm{MSE}}_{\mathcal{D}_I}\left(\sum_{i=1}^{I}\widehat{\mathrm{CDR}}_i(I+1)\right)$$

$$= \sum_{i=1}^{I}\widehat{\mathrm{MSE}}\left(\widehat{\mathrm{CDR}}_i(I+1)\right) + 2\cdot\sum_{i>k>0}\left(\widehat{\Psi}_{i,k}^{I} + \widehat{C}_{i,J}^{I}\cdot\widehat{C}_{k,J}^{I}\cdot\widehat{\Delta}_{k,J}^{I}\right)$$

where for $i > k > 1$

$$\widehat{\Psi}_{i,k}^{I} = \frac{\widehat{C}_{i,J}^{I}}{\widehat{C}_{k,J}^{I}}\cdot\left(1+\frac{\left(\widehat{\sigma}_{I-k}^{I}\right)^2/\left(\widehat{f}_{I-k}^{I}\right)^2}{S_{I-k}^{I+1}}\right)\cdot\left(1+\frac{\left(\widehat{\sigma}_{I-k}^{I}\right)^2/\left(\widehat{f}_{I-k}^{I}\right)^2}{C_{k,I-k}}\right)^{-1}\cdot\widehat{\Phi}_{k,J}^{I}$$

and $\widehat{\Psi}_{i,1}^{I} = 0$ *for* $i > 1$.

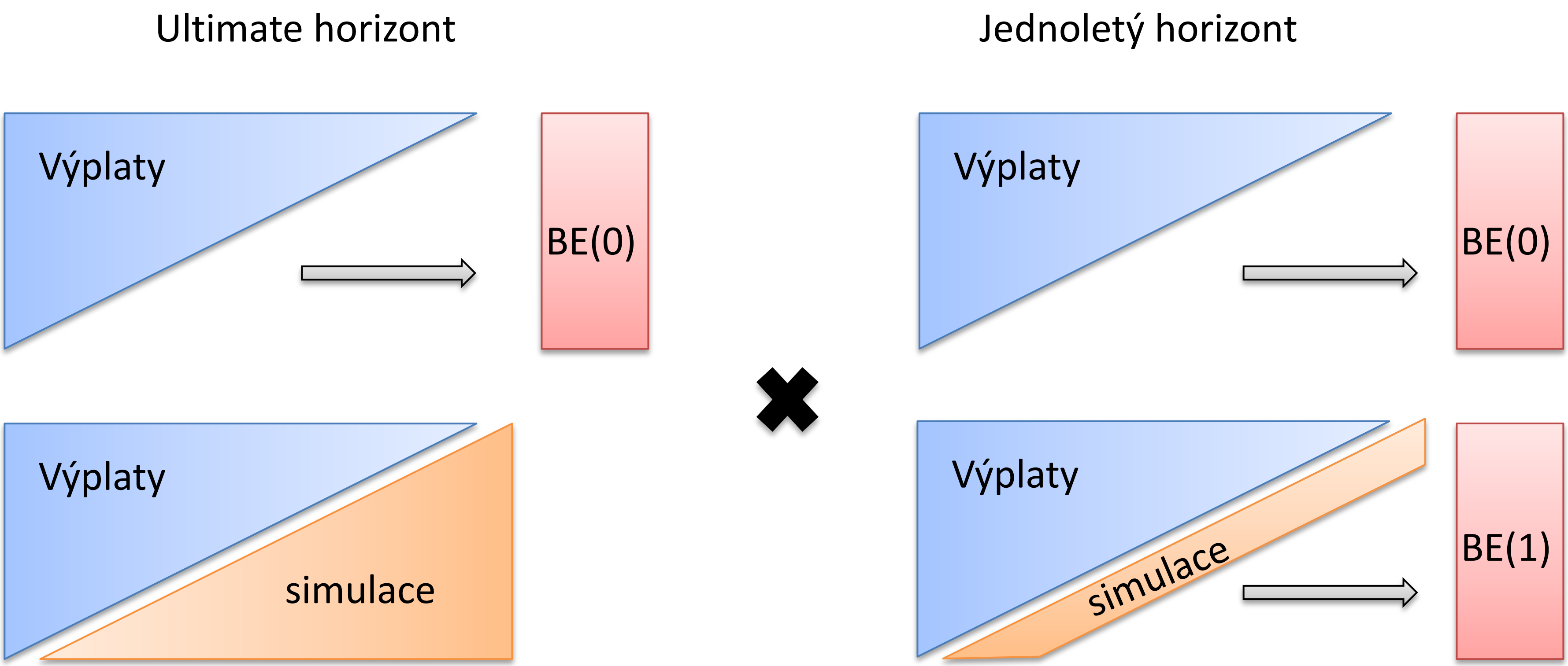
Stochastické modelování

Obecný princip:

1. Odhad budoucích pojistných plnění v čase t
2. Simulace pojistných plnění v intervalu (t,t+1]
3. Odhad budoucích pojistných plnění v čase t+1,
   se zohledněním nových (náhodně vygenerovaných) informací z intervalu (t,t+1]

Zřejmě nejčastěji uváděnou metodou je aplikace bootstrapu na vývojové trojúhelníky

Bootstrap ve vývojových trojúhelnících



Bootstrap ve vývojových trojúhelnících

Příprava:

1. Best estimate v čase 0
2. Fitování modelu - rekurzivní přepočet hodnot v horním trojúhelníku na základě odhadnutých vývojových faktorů
3. Výpočet reziduí

Simulace:

4. Vygenerování nového horního trojúhelníku („pseudo“)
5. Výpočet modelových hodnot nové diagonály pro pseudo data („mean prediction“)
6. Úprava hodnot nové diagonály o „process error“
7. Best estimte v čase 1

Ukázkový příklad bootstrap krok za krokem

- Data: smyšlená
- Metoda pro BE (a pro fitování trojúhelníků): Chain ladder
- Definice residuí: Adjusted Pearsons‘ Residuals

Vstupní data

Original cumulative triangle

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	520	1,230	2,750	2,820	3,110	3,310	3,310	3,810	3,810	3,870	4,070
2011-03	2011-03	60	770	1,120	2,120	2,120	2,120	3,120	3,120	4,020	4,020	
2011-04	2011-04	250	750	1,750	1,830	4,430	4,430	4,430	6,430	6,430		
2011-05	2011-05	0	360	1,410	1,530	2,200	2,300	2,300	2,300			
2011-06	2011-06	210	610	810	1,130	2,330	2,730	2,730				
2011-07	2011-07	1,000	1,050	1,650	2,100	2,100	2,100					
2011-08	2011-08	0	330	1,130	1,910	2,910						
2011-09	2011-09	220	220	720	1,020							
2011-10	2011-10	300	720	840								
2011-11	2011-11	210	630									
2011-12	2011-12	300										

Original incremental triangle

Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	520	710	1,520	70	290	200	0	500	0	60	200
2011-03	2011-03	60	710	350	1,000	0	0	1,000	0	900	0	
2011-04	2011-04	250	500	1,000	80	2,600	0	0	2,000	0		
2011-05	2011-05	0	360	1,050	120	670	100	0	0			
2011-06	2011-06	210	400	200	320	1,200	400	0				
2011-07	2011-07	1,000	50	600	450	0	0					
2011-08	2011-08	0	330	800	780	1,000						
2011-09	2011-09	220	0	500	300							
2011-10	2011-10	300	420	120								
2011-11	2011-11	210	420									
2011-12	2011-12	300										

Bootstrap – krok 1: Best estimate v čase 0

RESULTS								
Occurence period		No discounting						
PeriodStart	PeriodEnd	Latest incurred	Triangle ultimate	Triangle extrapolated	Tail value	Total extrapolated	Total ultimate
2011-02	2011-02	4,070	4,070	0	0	0	4,070
2011-03	2011-03	4,020	4,228	208	0	208	4,228
2011-04	2011-04	6,430	6,814	384	0	384	6,814
2011-05	2011-05	2,300	2,602	302	0	302	2,602
2011-06	2011-06	2,730	3,675	945	0	945	3,675
2011-07	2011-07	2,100	3,016	916	0	916	3,016
2011-08	2011-08	2,910	4,360	1,450	0	1,450	4,360
2011-09	2011-09	1,020	2,183	1,163	0	1,163	2,183
2011-10	2011-10	840	2,292	1,452	0	1,452	2,292
2011-11	2011-11	630	3,467	2,837	0	2,837	3,467
2011-12	2011-12	300	3,564	3,264	0	3,264	3,564
	Total	**27,350**	**40,271**	**12,921**	**0**	**12,921**	**40,271**

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Bootstrap – krok 2a: „Fitted cumulative triangle“

Chain ladder factors	0->1	1->2	2->3	3->4	4->5	5->6	6->7	7->8	8->9	9->10
	2.16	2.02	1.28	1.43	1.04	1.07	1.19	1.07	1.01	1.05

Fitted cumulative triangle

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	343	740	1,491	1,902	2,717	2,833	3,024	3,598	3,841	3,870	4,070
2011-03	2011-03	356	768	1,549	1,975	2,822	2,943	3,141	3,738	3,989	4,020	
2011-04	2011-04	574	1,238	2,497	3,184	4,548	4,744	5,062	6,024	6,430		
2011-05	2011-05	219	473	953	1,216	1,737	1,811	1,933	2,300			
2011-06	2011-06	309	668	1,346	1,717	2,453	2,558	2,730				
2011-07	2011-07	254	548	1,105	1,409	2,013	2,100					
2011-08	2011-08	367	792	1,597	2,037	2,910						
2011-09	2011-09	184	397	800	1,020							
2011-10	2011-10	193	417	840								
2011-11	2011-11	292	630									
2011-12	2011-12	300										

$$C_{i,J-i+1}^{fit} = C_{i,J-i+1}^{original}$$

$$C_{i,j}^{fit} = C_{i,j+1}^{fit} / \hat{f}_{j\to j+1}$$

Bootstrap – krok 2b: „Fitted incremental triangle“

Fitted incremental triangle												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	343	397	752	410	815	117	190	574	242	29	200
2011-03	2011-03	356	412	781	426	847	121	198	597	252	31	
2011-04	2011-04	574	665	1,259	687	1,365	195	319	962	406		
2011-05	2011-05	219	254	481	262	521	75	122	367			
2011-06	2011-06	309	358	679	370	736	105	172				
2011-07	2011-07	254	294	557	304	604	87					
2011-08	2011-08	367	425	805	440	873						
2011-09	2011-09	184	213	403	220							
2011-10	2011-10	193	224	423								
2011-11	2011-11	292	338									
2011-12	2011-12	300										

$$X_{i,1}^{fit} = C_{i,1}^{fit}$$

$$X_{i,j}^{fit} = C_{i,j+1}^{fit} - C_{i,j}^{fit}$$

Bootstrap – krok 3a: „Pearsons‘ residuals - unscaled“

Unscaled Pearson's residuals												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	9.6	15.7	28.0	-16.8	-18.4	7.7	-13.8	-3.1	-15.6	5.6	0.0
2011-03	2011-03	-15.7	14.7	-15.4	27.8	-29.1	-11.0	57.1	-24.4	40.9	-5.5	
2011-04	2011-04	-13.5	-6.4	-7.3	-23.2	33.4	-14.0	-17.8	33.5	-20.1		
2011-05	2011-05	-14.8	6.7	26.0	-8.8	6.5	2.9	-11.0	-19.2			
2011-06	2011-06	-5.6	2.2	-18.4	-2.6	17.1	28.7	-13.1				
2011-07	2011-07	46.8	-14.2	1.8	8.4	-24.6	-9.3					
2011-08	2011-08	-19.2	-4.6	-0.2	16.2	4.3						
2011-09	2011-09	2.7	-14.6	4.8	5.4							
2011-10	2011-10	7.7	13.1	-14.7								
2011-11	2011-11	-4.8	4.4									
2011-12	2011-12	0.0										

$$res_{i,j}^{unscaled} = \frac{X_{i,j}^{original} - X_{i,j}^{fit}}{\sqrt{X_{i,j}^{fit}}}$$

Bootstrap – krok 3b: „Pearsons‘ residuals - adjusted“

Adjusted Pearson's residuals												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	11.6	19.0	33.9	-20.3	-22.3	9.3	-16.7	-3.8	-18.9	6.8	0.0
2011-03	2011-03	-19.0	17.7	-18.7	33.7	-35.2	-13.3	69.1	-29.6	49.5	-6.7	
2011-04	2011-04	-16.4	-7.7	-8.8	-28.0	40.5	-16.9	-21.6	40.5	-24.4		
2011-05	2011-05	-17.9	8.1	31.5	-10.6	7.9	3.6	-13.4	-23.2			
2011-06	2011-06	-6.8	2.7	-22.3	-3.2	20.7	34.8	-15.9				
2011-07	2011-07	56.7	-17.2	2.2	10.1	-29.8	-11.3					
2011-08	2011-08	-23.2	-5.6	-0.2	19.7	5.2						
2011-09	2011-09	3.2	-17.7	5.8	6.5							
2011-10	2011-10	9.3	15.9	-17.9								
2011-11	2011-11	-5.8	5.4									
2011-12	2011-12	0.0										

$$res_{i,j}^{adjusted} = res_{i,j}^{unscaled} * scale\_factor$$

$$scale\_factor = \sqrt{\frac{\#observation}{degrees\_of\_freedom}} = \sqrt{\frac{66}{66-21}}$$

$$degrees\_of\_freedom = \#observations - \#parameters$$

Bootstrap – krok 3c: Residuals sample

1	11.6	23	-7.7	45	-15.9
2	19.0	24	-8.8	46	56.7
3	33.9	25	-28.0	47	-17.2
4	-20.3	26	40.5	48	2.2
5	-22.3	27	-16.9	49	10.1
6	9.3	28	-21.6	50	-29.8
7	-16.7	29	40.5	51	-11.3
8	-3.8	30	-24.4	52	-23.2
9	-18.9	31	-17.9	53	-5.6
10	6.8	32	8.1	54	-0.2
11	0.0	33	31.5	55	19.7
12	-19.0	34	-10.6	56	5.2
13	17.7	35	7.9	57	3.2
14	-18.7	36	3.6	58	-17.7
15	33.7	37	-13.4	59	5.8
16	-35.2	38	-23.2	60	6.5
17	-13.3	39	-6.8	61	9.3
18	69.1	40	2.7	62	15.9
19	-29.6	41	-22.3	63	-17.9
20	49.5	42	-3.2	64	-5.8
21	-6.7	43	20.7	65	5.4
22	-16.4	44	34.8	66	0.0

Bootstrap – krok 4a: Residuals resampling

Random item												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	54	52	35	34	30	41	44	20	46	7	48
2011-03	2011-03	40	13	14	8	38	18	4	40	36	11	
2011-04	2011-04	18	54	16	1	62	5	23	47	35		
2011-05	2011-05	46	2	34	19	2	62	55	17			
2011-06	2011-06	57	6	48	54	29	10	37				
2011-07	2011-07	1	62	49	53	50	59					
2011-08	2011-08	27	38	36	53	54						
2011-09	2011-09	48	49	20	40							
2011-10	2011-10	33	2	64								
2011-11	2011-11	37	46									
2011-12	2011-12	43										

1	11.6	23	-7.7	45	-15.9
2	19.0	24	-8.8	46	56.7
3	33.9	25	-28.0	47	-17.2
4	-20.3	26	40.5	48	2.2
5	-22.3	27	-16.9	49	10.1
6	9.3	28	-21.6	50	-29.8
7	-16.7	29	40.5	51	-11.3
8	-3.8	30	-24.4	52	-23.2
9	-18.9	31	-17.9	53	-5.6
10	6.8	32	8.1	54	-0.2
11	0.0	33	31.5	55	19.7
12	-19.0	34	-10.6	56	5.2
13	17.7	35	7.9	57	3.2
14	-18.7	36	3.6	58	-17.7
15	33.7	37	-13.4	59	5.8
16	-35.2	38	-23.2	60	6.5
17	-13.3	39	-6.8	61	9.3
18	69.1	40	2.7	62	15.9
19	-29.6	41	-22.3	63	-17.9
20	49.5	42	-3.2	64	-5.8
21	-6.7	43	20.7	65	5.4
22	-16.4	44	34.8	66	0.0

Resampled residuals												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	-0.2	-23.2	7.9	-10.6	-24.4	-22.3	34.8	49.5	56.7	-16.7	2.2
2011-03	2011-03	2.7	17.7	-18.7	-3.8	-23.2	69.1	-20.3	2.7	3.6	0.0	
2011-04	2011-04	69.1	-0.2	-35.2	11.6	15.9	-22.3	-7.7	-17.2	7.9		
2011-05	2011-05	56.7	19.0	-10.6	-29.6	19.0	15.9	19.7	-13.3			
2011-06	2011-06	3.2	9.3	2.2	-0.2	40.5	6.8	-13.4				
2011-07	2011-07	11.6	15.9	10.1	-5.6	-29.8	5.8					
2011-08	2011-08	-16.9	-23.2	3.6	-5.6	-0.2						
2011-09	2011-09	2.2	10.1	49.5	2.7							
2011-10	2011-10	31.5	19.0	-5.8								
2011-11	2011-11	-13.4	56.7									
2011-12	2011-12	20.7										

Bootstrap – krok 4b: „Incremental pseudo-data calculation“

Pseudodata incremental triangle												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	338	-65	969	195	119	-124	670	1,760	1,125	-61	231
2011-03	2011-03	406	773	259	349	171	882	-88	662	308	31	
2011-04	2011-04	2,229	659	9	991	1,952	-116	181	427	565		
2011-05	2011-05	1,058	557	247	-217	955	212	339	112			
2011-06	2011-06	366	535	736	366	1,836	175	-3				
2011-07	2011-07	439	567	796	207	-127	141					
2011-08	2011-08	43	-53	906	322	866						
2011-09	2011-09	214	361	1,397	260							
2011-10	2011-10	630	508	304								
2011-11	2011-11	64	1,381									
2011-12	2011-12	659										

$$X_{i,j}^{pseudo} = X_{i,j}^{fit} + res_{i,j}^{resampled} * \sqrt{X_{i,j}^{fit}}$$

Bootstrap – krok 4c: „Cumulative pseudo-data and development factors calculation“

Pseudodata cumulative triangle

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	338	273	1,242	1,437	1,555	1,431	2,101	3,861	4,986	4,925	5,156
2011-03	2011-03	406	1,179	1,438	1,786	1,958	2,840	2,752	3,413	3,722	3,752	
2011-04	2011-04	2,229	2,888	2,896	3,887	5,839	5,724	5,904	6,331	6,896		
2011-05	2011-05	1,058	1,615	1,862	1,645	2,601	2,813	3,151	3,263			
2011-06	2011-06	366	901	1,637	2,003	3,839	4,015	4,011				
2011-07	2011-07	439	1,006	1,802	2,009	1,881	2,022					
2011-08	2011-08	43	-11	896	1,218	2,084						
2011-09	2011-09	214	574	1,971	2,231							
2011-10	2011-10	630	1,138	1,442								
2011-11	2011-11	64	1,445									
2011-12	2011-12	659										

Pseudo data CH-L factors	0->1	1->2	2->3	3->4	4->5	5->6	6->7	7->8	8->9	9->10
	1.9025	1.58796	1.17988	1.41268	1.06626	1.06522	1.21285	1.1469	0.99648	1.04691

Bootstrap – krok 5a: „Mean prediction of next year diagonal“

Pseudo data CH-L factors	0->1	1->2	2->3	3->4	4->5	5->6	6->7	7->8	8->9	9->10
	1.9025	1.58796	1.17988	1.41268	1.06626	1.06522	1.21285	1.1469	0.99648	1.04691

Mean prediction cumulative triangle

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	338	273	1,242	1,437	1,555	1,431	2,101	3,861	4,986	4,915	5,156
2011-03	2011-03	406	1,179	1,438	1,786	1,958	2,840	2,752	3,413	3,722	3,752	**3,928**
2011-04	2011-04	2,229	2,888	2,896	3,887	5,839	5,724	5,904	6,311	6,896	**6,872**	
2011-05	2011-05	1,058	1,615	1,862	1,645	2,601	2,813	3,151	3,263	**3,742**		
2011-06	2011-06	366	901	1,637	2,003	3,839	4,015	4,011	**4,865**			
2011-07	2011-07	439	1,006	1,802	2,009	1,881	2,022	**2,154**				
2011-08	2011-08	43	-11	896	1,218	2,084	**2,222**					
2011-09	2011-09	214	574	1,971	2,231	**3,151**						
2011-10	2011-10	630	1,138	1,442	**1,702**							
2011-11	2011-11	64	1,445	**2,294**								
2011-12	2011-12	659	**1,254**									

Bootstrap – krok 5b: „Mean prediction incremental payments“

Mean prediction incremental triangle												
Occurence period		**Development period**										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02											
2011-03	2011-03											176
2011-04	2011-04										-24	
2011-05	2011-05									479		
2011-06	2011-06								854			
2011-07	2011-07							132				
2011-08	2011-08						138					
2011-09	2011-09					920						
2011-10	2011-10				259							
2011-11	2011-11			849								
2011-12	2011-12		595									

Bootstrap – krok 6a: „Residuals resampling for next year diagonal“

Random item

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02											
2011-03	2011-03											6
2011-04	2011-04										22	
2011-05	2011-05									11		
2011-06	2011-06								30			
2011-07	2011-07							3				
2011-08	2011-08						28					
2011-09	2011-09					65						
2011-10	2011-10				3							
2011-11	2011-11			42								
2011-12	2011-12		32									

1	11.6	23	-7.7	45	-15.9
2	19.0	24	-8.8	46	56.7
3	33.9	25	-28.0	47	-17.2
4	-20.3	26	40.5	48	2.2
5	-22.3	27	-16.9	49	10.1
6	9.3	28	-21.6	50	-29.8
7	-16.7	29	40.5	51	-11.3
8	-3.8	30	-24.4	52	-23.2
9	-18.9	31	-17.9	53	-5.6
10	6.8	32	8.1	54	-0.2
11	0.0	33	31.5	55	19.7
12	-19.0	34	-10.6	56	5.2
13	17.7	35	7.9	57	3.2
14	-18.7	36	3.6	58	-17.7
15	33.7	37	-13.4	59	5.8
16	-35.2	38	-23.2	60	6.5
17	-13.3	39	-6.8	61	9.3
18	69.1	40	2.7	62	15.9
19	-29.6	41	-22.3	63	-17.9
20	49.5	42	-3.2	64	-5.8
21	-6.7	43	20.7	65	5.4
22	-16.4	44	34.8	66	0.0

Resampled residuals

Occurence period		Development period										
PeriodStart	PeriodEnd	0->1	1->2	2->3	3->4	4->5	5->6	6->7	7->8	8->9	9->10	
2011-02	2011-02											
2011-03	2011-03											9.3
2011-04	2011-04										-16.4	
2011-05	2011-05									0.0		
2011-06	2011-06								-24.4			
2011-07	2011-07							33.9				
2011-08	2011-08						-21.6					
2011-09	2011-09					5.4						
2011-10	2011-10				33.9							
2011-11	2011-11			-3.2								
2011-12	2011-12		8.1									

Bootstrap – krok 6b: „Process error included prediction for next year“

Process error prediction

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02											
2011-03	2011-03											342
2011-04	2011-04										-66	
2011-05	2011-05									155		
2011-06	2011-06								-37			
2011-07	2011-07							544				
2011-08	2011-08						-117					
2011-09	2011-09					550						
2011-10	2011-10				747							
2011-11	2011-11			560								
2011-12	2011-12		498									

$$\sum_i X_i^{simulated} = 3{,}177$$

$$X_{i,j}^{simulated} = X_{i,j}^{mean\_prediction} + res_{i,j}^{resampled} * \sqrt{X_{i,j}^{mean\_prediction}}$$

Bootstrap – krok 7a: „Updated incremental triangle“

Updated incremental triangle

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	338	-65	969	195	119	-124	670	1,760	1,125	-61	231
2011-03	2011-03	406	773	259	349	171	882	-88	662	308	31	342
2011-04	2011-04	2,229	659	9	991	1,952	-116	181	427	565	-66	
2011-05	2011-05	1,058	557	247	-217	955	212	339	112	155		
2011-06	2011-06	366	535	736	366	1,836	175	-3	-37			
2011-07	2011-07	439	567	796	207	-127	141	544				
2011-08	2011-08	43	-53	906	322	866	-117					
2011-09	2011-09	214	361	1,397	260	550						
2011-10	2011-10	630	508	304	747							
2011-11	2011-11	64	1,381	560								
2011-12	2011-12	659	498									

Incremental pseudo-data triangle

&

Process error included prediction for next year

Bootstrap – krok 7b: „Updated cumulative triangle and Chain-ladder factors calculation“

Updated cumulative triangle

Occurence period		Development period										
PeriodStart	PeriodEnd	0	1	2	3	4	5	6	7	8	9	10
2011-02	2011-02	338	273	1,242	1,437	1,555	1,431	2,101	3,861	4,986	4,925	5,156
2011-03	2011-03	406	1,179	1,438	1,786	1,958	2,840	2,752	3,413	3,722	3,752	4,095
2011-04	2011-04	2,229	2,888	2,896	3,887	5,839	5,724	5,904	6,331	6,896	6,831	
2011-05	2011-05	1,058	1,615	1,862	1,645	2,601	2,813	3,151	3,263	3,418		
2011-06	2011-06	366	901	1,637	2,003	3,839	4,015	4,011	3,974			
2011-07	2011-07	439	1,006	1,802	2,009	1,881	2,022	2,566				
2011-08	2011-08	43	-11	896	1,218	2,084	1,967					
2011-09	2011-09	214	574	1,971	2,231	2,780						
2011-10	2011-10	630	1,138	1,442	2,189							
2011-11	2011-11	64	1,445	2,005								
2011-12	2011-12	659	1,157									

Updated data CH-L factors	0->1	1->2	2->3	3->4	4->5	5->6	6->7	7->8	8->9	9->10
	1.902195	1.379542	0.943284	1.073458	0.836181	0.860994	0.823432	0.748681	0.456156	0.332474

Updated data CH-L factors - cumulative	0->10	1->10	2->10	3->10	4->10	5->10	6->10	7->10	8->10	9->10
	8.298195	4.362432	2.747132	2.328353	1.648126	1.545626	1.451089	1.196452	1.043177	1.046904

Bootstrap – krok 7c: Best estimate v čase 1

RESULTS							
Occurence period		**No discounting**					
PeriodStart	**PeriodEnd**	Latest incurred	Triangle ultimate	Triangle extrapolated	Tail value	Total extrapolated	Total ultimate
2011-02	2011-02	5,156	5,156	0	0	0	5,156
2011-03	2011-03	4,095	4,095	0	0	0	4,095
2011-04	2011-04	6,831	7,151	320	0	320	7,151
2011-05	2011-05	3,418	3,566	148	0	148	3,566
2011-06	2011-06	3,974	4,755	781	0	781	4,755
2011-07	2011-07	2,566	3,723	1,157	0	1,157	3,723
2011-08	2011-08	1,967	3,040	1,073	0	1,073	3,040
2011-09	2011-09	2,780	4,582	1,802	0	1,802	4,582
2011-10	2011-10	2,189	5,097	2,908	0	2,908	5,097
2011-11	2011-11	2,005	5,508	3,503	0	3,503	5,508
2011-12	2011-12	1,157	5,047	3,890	0	3,890	5,047
	Total	**36,138**	**51,720**	**15,582**	**0**	**15,582**	**51,720**

Výstup

Krok 1: *BE(0)*

Opakováním kroků 4 až 7: empirické rozdělení *BE(1)* a *Claims(1)* získané ze simulovaných scénářů

$\Rightarrow$ nasimulované empirické rozdělení CDR

Bootstrap – pro a proti

Přínosy

- Stochastická metoda – výsledkem je kompletní empirické rozdělení CDR
- Velká variabilita ve volbě modelů pro výpočet BE

Omezení

- Časová náročnost – zejména při využití stochastického modelu pro výpočet BE

Další přístupy

Faktorový model

- Simulujeme pouze ultimate loss

$$CDR_i(I+1) = \hat{C}_{i,J}^{I} - \hat{C}_{i,J}^{I+1}$$

$$\hat{C}_{i,J}^{I+1} = \alpha * C_{i,J}^{simulace} + (1-\alpha) * \hat{C}_{i,J}^{I}$$

- Problematická může být kalibrace parametru α

Least Squares Monte Carlo simulations

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