Parameterisation, calibration and realisation of the full Internal Model

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Agenda

- Reasons for Internal model
  - Advantages/Disadvantages of the Internal model
  - How to build Internal Model
  - Calculation Engine structure

- Calculation engine modules
  - Dependencies
  - ESG & payment patterns
  - CAT risk
  - Gross UW
  - Outwards RI & RI Default
  - Reserve Risk
  - Gross Emergence
  - Other Risk modules
  - Financial Statements
  - Capital allocation

- Discussion & Questions

Reasons for Internal Model

- Capital requirement
  - Standard formula vs. Internal Model
- Use of the model
  - Regulatory requirements & Rating Agencies requirements
    - ICA, SCR/MCR, Solvency II to ultimate
  - Decision making process
  - Business planning
  - Underwriting, Reinsurance & Investment strategy testing
  - Capital allocation -> pricing
- Understanding of the business - Internal
  - Parameterisation and analysis consistency between departments (e.g. outwards reinsurance team, underwriters/pricing actuaries, capital modelling team)

Advantages/Disadvantages of Internal Model

+ Capital requirement

- Internal Model SCR 2-3 times lower than Standard Formula SCR

+ Use of the model

- Possibility to get higher rating with lower capital
- Reinsurance optimisation - possibility of reducing reinsurance costs
- Business planning - better view on business mix, manage the underwriting process, change in risk appetite
- Allow for capital allocation in pricing

+ Understanding of the business - Internal

- Parameterisation and analysis consistency between departments

- Costs, Resources & Time

- Software & Hardware costs, Consultancy
- Dedicated team of 2-4 people (Actuaries) + 2 people (Risk Management)
- Model build + documentation min of 12 months

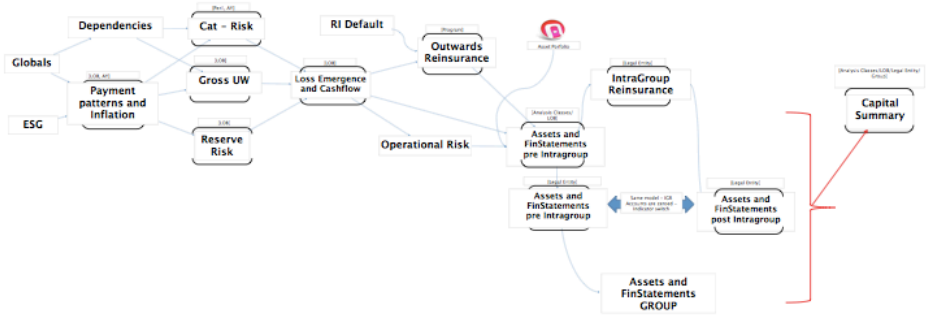
How to build the Internal Model

- What do we want?
  - SCR, ICA - Financial statements (incl. GAAP)
  - Risk category split (standalone / attribution)
  - Additional reports/analysis - based on user requirements

- Model design
  - Determine and design individual model components
    - Standard vs. non-traditional
    - Top->Down or Bottom->Up
  - Number of Simulations and Memory constraints
  - Importance of the run-time
    - Dependencies
    - Parallel x Serial
  - Scenario/Sensitivity testing intelligent design
    - Recalculate minimal number of model components

  - Model Map

Model Structure - example



Dependencies - general

- Dependency types
  - Gaussian copula, Student-T copula
  - Archimedean copula (Clayton, Gumbel, Frank) - example
- Dependency groups
  - Within Risk Categories
    - UW risk (Classes of business, attritional x large losses)
    - Reserve Risk (Classes of business, adjacent years)
    - CAT Risk (Similar type of Catastrophes)
    - Credit Risk (Reinsurers)
    - Operational Risk (Scenarios within same category)
  - Between Risk Categories
    - Market Risk & Cat Risk (short tail CATs - Man made & Natural)
    - UW risk x Reserve risk
    - Cat risk x Credit risk
    - Operational Risk x Insurance risks

Dependencies - parameterisation

- Estimate using data
  - Problematic parameterisation due to lack of data
- Expert Judgement
  - Low x Medium x High - symmetric and positive semidefinite matrix
    - Archimedean Copulas parameter (e.g. Gumbel > 1)
  - Adjusting matrix to be positive semidefinite - example

$$C = \begin{pmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.3 \\ 0.7 & 0.3 & 1 \end{pmatrix}$$

$$\{\, 2.2967 \quad 0.710625 \quad -0.00735 \,\}$$

$$C = \begin{pmatrix} 1 & 0.89458 & 0.69662 \\ 0.89458 & 1 & 0.30254 \\ 0.69662 & 0.30254 & 1 \end{pmatrix}$$

$$\{\, 2.2967 \quad 0.710625 \quad 0 \,\}$$

  - Sensitivity testing (decrease/increase by 10%)

ESG & Payment Patterns and Inflation

- ESG - External model
  - Link to Internal model
  - Exchange rate calculation
  - Inflation weights - Class specific

$$Inflation_{t,ClassA} = \alpha_{ClassA} Inf_{t,Price} + (1 - \alpha_{ClassA}) Inf_{t,Wage} + SuperimposedInf$$

- Payment patterns
  - Stochastic decay patterns
    - Pct of reserve, sum always 1 - example *.xls*
  - Common assumption - Mid year payments
  - Inflation adjusted payment patterns

- Calculation of Ultimate factors/rates
  - “Ultimate” Inflation factors
  - “Ultimate” Exchange rates
  - “Ultimate” Discount factor

Catastrophic Risk

- RMS/AIR (externally modelled) Perils x Other Perils
  - Definition (Loss in excess of x mil USD, high RP)
  - Natural Catastrophes vs Man-Made, Short tail vs Long tail
- Modelling of RMS/AIR Perils
  - Natural catastrophes (earthquake, windstorm)
  - Loss to company and market losses (predominant currency)
  - Loss dates
  - 1st approach: ELTs - example
  - 2nd approach: Pre-simulated values from external models
- Modelling of “other” Perils
  - Man-made (Terrorism, nuclear, medical procedures) and non-RMS modelled perils (Australia/Thai flood etc.)
  - Frequency per Peril x aggregated loss by each class
- Modelling extras
  - Modelled on AY basis and then allocated to UW years - applying exposure weighted scaling factor

Catastrophic Risk - parameterisation

- RMS/AIR (externally modelled) Perils
  - Produced by Aggregate modelling team - on AY basis
  - 1st approach: Import the portfolio and produce the ELTs
  - 2nd approach: Import the portfolio and Pre-simulate the losses

- Non-RMS/AIR Perils
  - Parameterised on AY basis
  - Lack and low data quality
  - Setting the mean return period for each peril (>RP 25/10 years)
  - Selection of different RP and estimation of corresponding max loss to class
    - Fitting the distribution - example

Gross UW Risk

- Non-Cat Losses
  - Attritional losses - aggregate distribution
  - Large losses - frequency x severity
  - Loss Dates & UW Dates
  - Modelled on AYs or UW Years and allocated down to UW Years or AYs
    - Allocation - stochastic assignation or deterministic split
- Expenses & Premium
  - Acquisition, operating, fixed expenses
  - Gross Premium
  - Dominant currency
- Parameterisation & Calibration
  - Analysis of historical losses (MM or MLE)
  - Large losses threshold and cap
  - Calibration to business plan (AY or UW Year basis)
    - Roll forward by change in exposure

Outwards Reinsurance & RI Default

- Traditional Programmes
  - QS, XoL, SL, Cat Aggs, Fac
  - Loss ordering (Net of previous programmes) & recovery cashflows
  - Modelled functionalities (LOD x RAD, contractual exchange rates, program currency, programmes orderings, payments lags, event caps, term/event limits, ceding commission, profit commission)
- non-Traditional Programmes
  - ILW, CILW, Cat-bonds, Franchise, Reverse Franchise
- RI Default
  - Rating by Reinsurer, default rate & loss given default
  - Credit migration
  - Next year programmes placements
- Parameterisation
  - Simplifications

Reserve Risk

- Gross Reserve
  - Aggregate distribution by Historical UW Year or AY
  - Deterministic split to different currencies or dominant currency
- Net Reserve
  - Gross to Net ratio - vary by percentile
  - Additionally there are imported Gross and Net paid to date losses
    - ADC (LPT) modelling purposes
    - Intra-group reinsurance modelling purposes (Stop-Loss/QS covers)
- Historical premium & expenses
  - Modelling future payments/receivables/balances of opening outstandings
- Reserve Perils
  - Frequency x Severity
  - Factor of reserves

Reserve Risk - parameterisation

- Gross Reserve Risk
  - Bootstrap
    - Frequently used approach to get some initial estimate of volatility
    - Simple and flexible approach
  - Mack & Munich Chain Ladder
  - Actuarial Hi-Low
    - Changing of assumptions to estimate level of volatility
  - Gross Reserve modelled by LogNormal distribution
- Net Reserve Risk
  - Modelled by Gross to Net Ratio
  - These could vary by different percentiles to allow for non-proportional reinsurance contracts

Gross Emergence

- Gross Loss Emergence
  - Complete Re-Reserving in model - not recommended
  - Perfect view on ultimate - for some risks
  - Chain ladder development factors and/or BF method
  - Factor approach (weights between mean and true ultimate) - preferred approach
    - Simple and easy to validate

$$EX + \alpha(X - EX) = (1 - \alpha)EX + \alpha X$$

- RI Recoveries Emergence
  - Calculate recoveries for 1 year gross views
    - Same proportion between emergence and ultimate for all losses
  - Apply the gross loss emergence factors to ultimate RI recoveries
    - Limitations (ART, non-proportional contracts)

Other Models

- Operational Risk
  - Market, Credit & Insurance scenarios
  - cca 10% of capital
  - Frequency x Severity approach by scenario
- IntraGroup Reinsurance
  - QS, SL
  - Similar as outwards reinsurance modelling

Assets and Financial Statements

- Solvency II (1 year)
  - Balance sheet (opening & closing)
  - P&L statement
  - SCR & MCR
  - Functional currency
- GAAP (1 year)
  - Balance sheet (opening & closing)
  - P&L statement
  - Functional currency
- ICA (to ultimate)
- Additional modelled risks
  - Broker balances
  - Liquidity risk (selling asset portfolio, borrow at penal rate)
  - Asset portfolio rebalancing (duration and/or currency match, minimal holdings per Asset class)
    - No transaction costs
- Examples

Capital allocation

- Allocation
  - Risk categories - example
    - Insurance Risk
      - UW Risk - Cat risk x non-Cat risk
      - Reserve Risk
    - Market Risk
      - FX risk
      - Spread risk
      - Interest risk
      - Default risk
    - Operational Risk
  - Classes of business
- Allocation techniques
  - co-TVaR
  - Shapley (last-in, first-in)
  - Allocation by standalone risk profile
  - example

Questions & Comments

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Thank you

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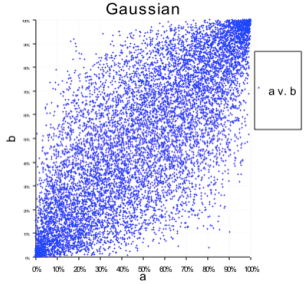
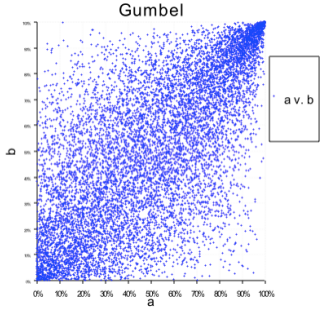
Dependencies - example 1

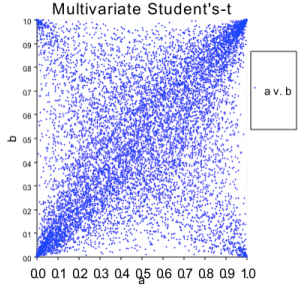
Gaussian



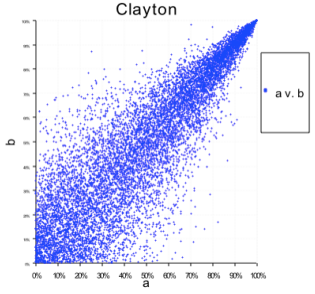
Gumbel



Multivariate Student's-t



Clayton



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Dependencies - example 2

- Method 1
  - Formula $M' = \frac{1}{1-\lambda_{min}}(M - I\lambda_{min})$

$$M = \begin{pmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.3 \\ 0.7 & 0.3 & 1 \end{pmatrix} \quad \{\, 2.2967 \quad 0.710625 \quad -0.00735 \,\}$$

$$\Rightarrow \quad M' = \begin{pmatrix} 1 & 0{,}8934 & 0{,}6949 \\ 0{,}8934 & 1 & 0{,}2978 \\ 0{,}6949 & 0{,}2978 & 1 \end{pmatrix} \quad \{\, 2{,}287 \quad 0{,}713 \quad 0 \,\}$$

- Method 2
  - Calculate eigenvalues and right hand side eigenvectors of M
  - Set all negative eigenvalues to 0
  - Set the length of the eigenvector to its associated eigenvalue
  - Arrange the eigenvectors as the columns of the matrix C
  - C' results from C by normalising the row vectors of C to unit length
  - Calculate $M'=C'C'^{T}$

$$C = \begin{pmatrix} 1 & 0.9 & 0.7 \\ 0.9 & 1 & 0.3 \\ 0.7 & 0.3 & 1 \end{pmatrix} \quad \{\, 2.2967 \quad 0.710625 \quad -0.00735 \,\}$$

$$\Rightarrow \quad C = \begin{pmatrix} 1 & 0.89458 & 0.69662 \\ 0.89458 & 1 & 0.30254 \\ 0.69662 & 0.30254 & 1 \end{pmatrix} \quad \{\, 2.2967 \quad 0.710625 \quad 0 \,\}$$

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Catastrophic Risk - example 1

- Simulations using ELT

EVENT ID	RATE	PERSPVALUE	STDEVI	STDEVC	EXPVALUE
151076	0,00897	15629,0	24387,9	48775,8	97551,6
158525	0,00913	50150,5	19561,2	39122,5	78245,0
171773	0,00162	11337,0	21910,0	43820,1	87640,1
114034	0,00999	10618,4	9328,3	18656,6	37313,2
185157	0,00826	589,0	243,7	487,4	974,7
...	...	...	...	...	...
167638	0,00473	5443,0	13011,5	26022,9	52045,9

  - Frequency - Poisson
  - Secondary uncertainty - Damage Ratio (Beta distribution)

$$\alpha = \left(\frac{EX(1-EX)}{VarX} - 1\right) EX \qquad EX = PERSPVALUE$$

$$\beta = \left(\frac{EX(1-EX)}{VarX} - 1\right)(1-EX) \qquad VarX = \left(\frac{STDEVI + STDEVC}{2}\right)^2$$

  - Loss = EXPVALUE*Damage Ratio

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Catastrophic Risk - example 2

- non-RMS/AIR parameterisation

Name of Peril	PML 1	RP 1	PML 2	RP 2	Max Loss
Flood Australia	5 000 000	25	15 000 000	250	22 000 000

- Frequency - Poisson $\lambda = \frac{1}{RP1}$
- Severity - Pareto

$$F(x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha} ; \alpha > 0, \beta > 0, x > \beta$$

$$\beta = PML1$$

$$\alpha = \ln\left(\frac{-\ln\left(1 - \frac{1}{RP2}\right)}{\lambda}\right) \div \ln\left(\frac{PML2}{PML1}\right) = 2.094$$

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Financial Statements - example 1

	UK GAAP Balance Sheet	
ASSETS	Year – Closing	Year – Opening
Government Bonds	0	0
Corporate Bonds	0	0
Equity	0	0
Other Investments & Derivatives	0	0
TOTAL INVESTMENTS	0	0
Goodwill	0	0
Intangible Assets	0	0
Cash	0	0
Premiums Receivable	0	0
Intermediary Receivable		
Deferred Tax Assets	0	0
Deferred Gross Acquisition Costs	0	0
Reinsurance Recoverables	0	0
RI Share of Unearned Premiums	0	0
Other Assets	0	0
TOTAL ASSETS	0	0

LIABILITIES AND SHAREHOLDERS' EQUITY	Year – Closing	Year – Opening
Gross Loss Reserve	0	0
Gross Unearned Premiums	0	0
Accrued Underwriting Expenses	0	0
Reinsurance Payables	0	0
Funds	0	0
Other Indebtedness	0	0
Subordinated Debt	0	0
Deferred Tax Liability	0	0
RI Share of Deferred Acquisition Costs	0	0
Other Liabilities	0	0
TOTAL LIABILITIES	0	0
Capital	0	0
Retained Earnings	0	0
Accumulated Other Comprehensive Loss	0	0
TOTAL SHAREHOLDERS EQUITY	0	0
TOTAL LIABILITIES AND SHAREHOLDERS EQUITY	0	0

	Solvency II Balance Sheet	
ASSETS	Year – Closing	Year – Opening
Government Bonds	0	0
Corporate Bonds	0	0
Equity	0	0
Other Investments & Derivatives	0	0
TOTAL INVESTMENTS	0	0
Cash	0	0
RI Share of Claims Provision	0	0
RI Share of Premium Provision	0	0
Other Assets	0	0
TOTAL ASSETS	0	0

LIABILITIES AND SHAREHOLDERS' EQUITY	Year – Closing	Year – Opening
Gross Claims Provision	0	0
Gross Premium Provision	0	0
Risk Margin	0	0
Other Liabilities	0	0
TOTAL LIABILITIES	0	0
Capital	0	0
Retained Earnings	0	0
Accumulated Other Comprehensive Loss	0	0
TOTAL SHAREHOLDERS EQUITY	0	0
TOTAL LIABILITIES AND SHAREHOLDERS EQUITY	0	0

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UK GAAP P&L STATEMENT	Year
WRITTEN PREMIUMS	
Gross Written Premiums	0
Inwards Reinstatements	0
RI Written Premiums	0
Outwards Reinstatements	0
NET Written PREMIUM	0
EARNED PREMIUMS	
Gross Earned Premiums	0
Inwards Reinstatements	0
RI Earned Premiums	0
Outwards Reinstatement	0
TOTAL EARNED PREMIUM	0
EXPENSES	
Gross Acquisition Costs Incurred	0
RI Acquisition Costs Incurred	0
Operating Costs Incurred	0
Inwards RI Commission Incurred	0
Outwards RI Commission Incurred	0
TOTAL NET EXPENSES	0
LOSSES	
Gross Losses Incurred	0
RI Recoveries Incurred	0
TOTAL NET LOSSES	0
UNDERWRITING PROFIT	0
OTHER REVENUE	
Net Investment Income	0
Realized Gains on Investments	0
TOTAL OTHER REVENUE	0
OPERATING PROFIT	0
OTHER ACCOUNTS	
Interest Expense	0
Provision for Income Taxes	0
PRE TAX PROFIT	0

ECONOMIC P&L STATEMENT	Year
PAID PREMIUMS	
Gross Received Premium	0
Change in Gross Premium Provision	0
Paid RI Premium	0
Change in RI Share of Premium Provision	0
TOTAL NET PAID PREMIUM	0
PAID EXPENSES	
Gross Acquisition Costs Paid	0
RI Acquisition Costs Paid	0
Operating Costs Paid	0
Inwards RI Commission Paid	0
Outwards RI Commission Paid	0
TOTAL NET PAID EXPENSES	0
LOSSES	
Gross Losses Incurred	0
RI Recoveries Incurred	0
Change in Market Value Margin	0
TOTAL NET LOSSES	0
UNDERWRITING PROFIT	0
OTHER REVENUE	
Net Investment Income	0
Realized and Unrealized Gains on Investments	0
TOTAL OTHER REVENUE	0
OPERATING PROFIT	0
OTHER ACCOUNTS	
Interest Expense	0
Provision for Income Taxes	0
PRE TAX PROFIT	0

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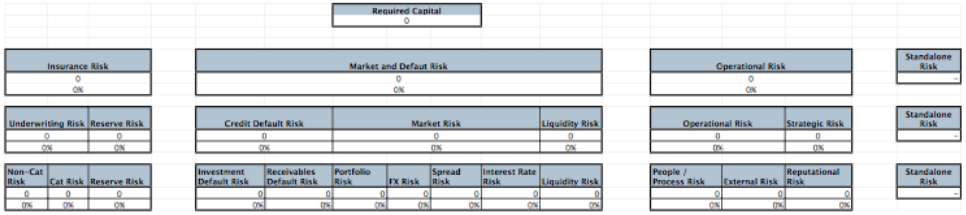
Capital Allocations - example 1



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Capital Allocations - example 2

Input		Capital Calcs		Capital Allocation			
Class of Business	STD	Multiplier	Standalone Capital	co-TVaR	First-in (Standalone)	Last-in	Shapley
A	429	2,33	1 000	117	548	163	345
B	858	2,33	2 000	763	1 095	668	906
C	1 288	2,33	3 000	1 649	1 643	1 580	1 654
D	1 717	2,33	4 000	2 949	2 191	3 066	2 572
Aggregated	2 351		5 477				

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